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PRACTICAL PERSPECTIVE.

BEING

A COURSE OF LESSONS,

EXHIBITING

Eaſy and conciſe RULES for drawing juſtly all Sorts of Objects.

Adapted to the USE of SCHOOLS.

By H. CLARKE.

k.

IN TWO VOLUMES.

VOL. I.

Sempre la pratica di pittura deve eſſere edificata ſopra la buona teorica, della quale la proſpettiva è guida, e porta: e ſenza quella niente ſi fà bene, coſì di pittura, come in ogn'altra profeſſione.
Lionardo da Vinci della Pittura. Cap. XXIII.

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M DCC LXXVI.

G R III
BRITISH MUSEUM

TO

CHARLES WHITE, Eſq;

OF MANCHESTER,

FELLOW OF THE ROYAL SOCIETY.

SIR,

AS your extenſive Knowledge and good Taſte command the Eſteem of all that profeſs or love the Liberal Arts and Sciences, I have thence preſumed to addreſs the following Sheets to you; being well aſſured, that if they ſhall have the good Fortune to meet with your Approbation, I need not be anxious about their Reception elſewhere.

Were

DEDICATION.

Were your Character, Sir, leſs diſtinguiſhed, the Obligations I am under to your Goodneſs, would, alone, be Motives ſufficient to make me gladly embrace this Opportunity of publicly expreſſing my warmeſt Gratitude, and of teſtifying the perfect Eſteem with which I am,

SIR,

Your moſt obliged

Humble Servant,

HENRY CLARKE.

PREFACE.

IT may, perhaps, have the Appearance of Vanity to attempt to write upon a Subject which ſeems already to have been quite exhauſted. But as the Utility of Perſpective begins now to appear to every one, and is no longer looked upon merely as a genteel Accompliſhment, but as a moſt neceſſary Acquiſition; I preſume, an Attempt to render this agreeable Art more eaſy and intelligible to the Learner, will not be thought impertinent. Indeed, I do not pretend to offer any Thing new in Principle; but the Method I have obſerved, I flatter myſelf, will be found to be better adapted to the Capacities of Youth, than what is propoſed in any Treatiſe that has hitherto appeared on the Subject. In order to be well informed in this Particular, I have been at the Trouble of collecting all the different Treatiſes of Perſpective I could hear of, both in our own and other Languages; and from a Compariſon it appears, that while the one Part have written purely for Mathematicians, the other have treated the Subject in ſo limited a Manner, that the Reader could not from thence acquire the leaſt Idea of the Principles of the Art he was endeavouring to learn.

The Principle, upon which this Method of Perſpective is founded, is ſo obvious to the Underſtanding, that it is a Matter of Surprize, it has not been univerſally received by the Connoiſſeurs. For what can be more eaſy to conceive, than, if a Plane of Glaſs be ſet betwixt our Eye and an Object, that the Rays, in paſſing from the Object to the Eye, will, by their Interſection with the Glaſs, there exhibit an exact Repreſentation of whatever the Eye is directed to. The Buſineſs then of this Method of Perſpective (and it is the only rational one) appears to be nothing more than the finding of thoſe Points of Interſection. This is done in Leſſon I. in ſo plain and ſimple a Manner, that the Method of Operation is known by a bare Inſpection. Indeed, the Demonſtration given to this fundamental Propoſition, may not be comprehended by every one; but, perhaps, to ſuch it may be thought a ſufficient Proof of the Truth of the Operation, to find, that when the Planes are all brought into one, or extended on the Ground Plane, the Thread, or Viſual Ray, ſtill interſects the Picture in the ſame Point, in a ſtraight Line to the Eye.

When the firſt Leſſon is clearly underſtood, the main Point is accompliſhed. For as Lines, Planes, and Solids, of whatever Figure, may be eaſily conceived to conſiſt of Points, the Perſpective of them may be found by this one general Rule. This alone I have often found ſufficient to ſeveral grown Perſons, to let them into the whole Practice of Perſpective. But (as Mr. Locke obſerves) ſince we always find the Minds of Youth are opened by Degrees, and that it is Exerciſe alone which opens them, it is neceſſary for them to go through a regular Courſe of Leſſons, in order to ſee the Application of this Rule to different Objects. Having therefore ſhown the Method of finding the Perſpective of a Point, I have proceeded to Lines and Planes ſituated on, and parallel to, the Ground Plane; then have gone to Lines, Planes, and Solids perpendicular to that Plane; and then ſhown the Perſpective of Lines, Planes, and Solids inclined to the ſame Plane. And in this laſt I have been the more particular, as it is a Part of Perſpective which is ſeldom underſtood. I do not know any that has rendered it practical, but Kirby. I have thence proceeded to Sciagraphic Perſpective, which, as it is a very eſſential Part in Drawing, I have conſidered every Way, both from the Sun and Candle. The Rules for Catoptric Perſpective are next given, when the reflecting Surface is parallel, perpendicular, and inclined to the Horizon. I have then exhibited the Nature of Theatrical Perſpective; and alſo ſhown the Theory of Horizontal Pictures, or Ceiling Pieces. And have concluded the firſt Volume, with a a practical Method of taking Perſpective Views without the Aſſiſtance of any Inſtrument.

In the ſecond Volume I have ſhown the Application of the foregoing Rules to a Variety of Subjects; as in Architecture, to the Perſpective Elevations of the Orders, Arches, Doors, Niches, &c. with the Perſpective of various Modes of Buildings, Squares, Streets, Avenues, &c. in Geography, to the Projections of the Sphere, for the Conſtruction of Maps, Charts,

&c.

&c. in Aſtronomy, to the Conſtruction of Celeſtial Planiſpheres, Analemmas, Aſtrolabas, Dials, &c. as alſo to the Conſtruction of Tranſits, Solar and Lunar Eclipſes, &c. I have alſo tranſlated from that excellent Italian Piece, *Trattato della Pittura di Lionardo da Vinci*, his Obſervations on the *Keeping of Ærial Perſpective.* And have concluded the Whole with ſome uſeful Hints to Painters for the Mixing of Colours, on the Principle of Sir Iſaac Newton's Theory. The inverſe Method of Perſpective I have omitted, as being of little or no Uſe; for any Perſon may find the vaniſhing Line of a Perſpective View, by producing any two Lines which are perſpectively parallel, till they interſect each other; for that Point of Interſection will always be in the vaniſhing Line. Neither have I ſaid any Thing of the Mechanical Methods of taking Views, copying Prints, &c. as thinking theſe beneath the Attention of the real Artiſt, who is ſeldom inclined to have Recourſe to ſuch Helps. A Camera Obſcura, indeed, is ſometimes uſeful in taking the Perſpective View of a Town; as it is not a very eaſy Matter to draw ſuch a Number of Objects all in true Proportion, merely by the Eye. But when the View is not crowded, nor very extenſive, the Method I have before laid down ſhould always be preferred. In the Catoptric Perſpective I might have ſhown the Principles of the Anamorphoſis, and their Rectification; but as this hath not an immediate Relation to the common Practice of Perſpective, I thought it would be rather leading the Learner out of his Way, and have therefore omitted it alſo.

Such is the Plan of this Performance: Which, however, I do not pretend to ſay hath no Imperfections, for *Humanum eſt errare.* But this I will venture to affirm, that though it is as conciſe as poſſible, yet it is as complete as neceſſary, either for the School or private Study. I have alſo been particularly careful to have it correct in the Printing, having frequently obſerved, that when a Work of this Kind appears in the World, the greater Part of thoſe who ſet up for Judges, are ſo bent to depreciate every Thing to which they can frame the leaſt Pretence of an Objection, that a ſingle typographical Error would be ſufficient for them to paſs Sentence on the whole Performance.

It were well that thoſe who are ambitious of paſſing for Connoiſſeurs in Drawing, would make themſelves perfectly acquainted with the Principles of this Art. And then we ſhould not ſo often meet with thoſe unjuſt Criticiſms, theſe Gentlemen are ſo apt to throw out indiſcriminately without regarding the Prejudice they may do a Perſon's Reputation, with thoſe who may have an high Opinion of their aſſumed Knowledge. It is in this, indeed, as in moſt other Sciences, that as ſoon as ſome Perſons have got a Smattering therein, they immediately ſet up for profeſſed Adepts. But as ſuch muſt be ſenſible of their own Deficiency, (otherwiſe their Caſe is pitiable indeed) they ſhould be cautious how they advance, leſt they ſhould not be furniſhed with a good Retreat. The Jeſuit's Perſpective has made more Pretenders to this Science, than any other Book on the Subject. It is a mere Jumble of Practice without Theory, and full of Abſurdities, and is therefore a very improper Book for Learners:

Decipit exemplar vitiis imitabile.———

Nothing is more common with this Author (and all thoſe who have copied from him) than to give the Perſpective of an Houſe, &c. with one Side parallel to the Line of Interſection, and the other diminiſhing into the Centre of the Picture. Indeed there is nothing wrong in this Method, if the Views are accounted only partial ones; but when the View of a Gentleman's Seat, &c. which makes the whole Picture, is exhibited by this Method, there is nothing more abſurd. It is juſt as if a Painter ſhould ſhow you a fine Portrait with Half the Face covered, and then leave you to judge of the Likeneſs from that. For as the Centre of the Picture is always oppoſite to the Spectator's Eye, is it not plain, by this Method, that the Houſe, &c. which ſhould be the principal Object of his View, is only a Part of a general One, and that he directs his Eye to one Object, in order to look at another? And is it not equally as evident, that unleſs the Obſerver be ſituated exactly in the Front of an Houſe, it will be always viewed on the Angle, and muſt therefore have two Points of Diminution? How ridiculous then muſt a Perſon, who has no better Notions of *linear* Drawing than what the Jeſuit's Perſpective

ſpective affords him, render himſelf to the real Connoiſſeur, by calling this *bad Perſpective*, when, in Fact, there is no other Method right.

I have been naturally led into theſe Reflections, by being more than once informed of the ill-natured Criticiſms, on a Perſpective View which I lately took of a Gentleman's Houſe near Mancheſter, by a Perſon who paſſes for a Connoiſſeur in Drawing; and thoſe Criticiſms made merely becauſe it has two Points of Diminution! *Riſum teneatis?* Whether the Defect lies in this Gentleman's Heart or Eye, I ſhall not pretend to determine, but will charitably ſuppoſe the latter. Perhaps the Perforation of the Uvea is not ſituated exactly in the Axis of his Eye. If ſo, the Rays of Light, after Refraction by the Cryſtaline and Vitreous Humours, will fall obliquely on the Expanſion of the Optic Nerve in the Fundus, and thereby cauſe an Obliquity of Viſion. In ſuch a Caſe, his Eye may appear to be directed to one Object, and at the ſame Time the Retina be impreſſed with the Image of another; and therefore his *Point of Sight* may be out of the Picture, and yet the Central Ray thereof coincide with the Axis of his Eye. The Effect is obvious. The moſt juſt Repreſentation to ſuch an Eye, muſt appear to be a mere *Anamorphoſis.*

Another Abſurdity in the common Practice of Perſpective, which it may not be amiſs juſt to take Notice of, eſpecially, ſince even Pozzo himſelf has fallen into the Miſtake, is the Reducing of Objects of equal Magnitudes, which are elevated above the Centre of the Picture, according to the Angle they ſubtend at the Spectator's Eye. This will immediately appear erroneous by a bare View of Fig. 4. For, if 1, 2, 3, 4, be Points equally diſtant from each other on the original Plane O R, and P E be the Perſpective Plane, and S the Eye; it is plain, that the Points of Interſection 1, 2, 3, 4, by the viſual Rays 1 S, 2 S. &c. will be at equal Diſtances from one another, becauſe P E is parallel to O R, and therefore no Reduction is neceſſary.

A great many more Abſurdities might be pointed out in the Practice of Perſpective, through proceeding on wrong Principles. But as my Intention is not ſo much to turn Critic on others, as to avoid the Laſh of Criticiſm myſelf, I ſhall ſay no more on that Point; being well aſſured, that Excellence in Writing upon any Art or Science, depends not on the Expertneſs of detecting the Miſtakes of others, but on Docility and good Senſe: Which is, in Part, agreeable to the Sentiment of Horace,

Scribendi recte, ſapere eſt et principium et fons.

Commercial *and* Mathematical School,
Salford, *near* Mancheſter, *June* 1, 1776.

PRACTICAL

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PRACTICAL PERSPECTIVE.

DEFINITIONS.

DEF. I.

PERSPECTIVE, is the art of deſcribing on a plane the exact repreſentation of any object.

DEF. II.

The *Original Plane*, is that plane on which an object is ſituated. If it be parallel to the horizon, and that on which the ſpectator ſtands, it is called the *Ground Plane*.

DEF. III.

The *Perſpective Plane*, or *Picture*, is a tranſparent plane, which is imagined to ſtand perpendicular to the ground plane, between the ſpectator's eye and an object.

DEF. IV.

The *Point of Sight*, is that point where the ſpectator's eye is placed to look upon the picture.

DEF. V.

The point where the picture is cut by a perpendicular line drawn from the point of ſight, is called the *Centre of the Picture*. And the diſtance between this point and the point of ſight, is called the *Diſtance of the Picture*.

DEF. VI.

The *Interſection of an Original Plane*, is the line where that original plane continued cuts the picture. And the *Interſection of an Original Line*, is the point where that original line continued cuts the picture.

DEF. VII.

A plane paſſing through the point of ſight parallel to an original plane, is called the *Parallel* of that plane. And a line paſſing through the point of ſight parallel to an original line, is called the *Parallel* of that line.

DEF. VIII.

The *Vaniſhing Line* of a plane, and the *Vaniſhing Point* of a line, are the interſections of their reſpective parallels, with the perſpective plane, or picture.

DEF. IX.

A line drawn from the point of ſight perpendicular to a vaniſhing line, is called the *Diſtance* of that vaniſhing line. And the point where the vaniſhing line is cut by the perpendicular, is called the *Centre* of that vaniſhing line. The diſtance between the vaniſhing point of a line and the point of ſight, is alſo called the *Diſtance* of that vaniſhing point.

DEF. X.

The point where a perpendicular, drawn from an original point, meets the line of interſection, is called ſimply the *Seat* of that point. And if the original point be ſituated on an inclined plane, the point where a perpendicular meets the ground plane, is called the *Ground Seat* of that point.

DEF. XI.

A line drawn from the ſeat of a point to the centre of the picture, is called the *Directing Line*,

Line, or *Director* of that point. And a line drawn from the interſection of a line to its vaniſhing point is alſo called the *Director* of that line. If the original line be ſituated on the ground plane, its directing line is called a *Primary Director*; if on an inclined plane, a *Secondary Director*.

DEF. XII.

The *Viſual Ray*, is a ſtraight line, proceeding from the object to the ſpectator's eye.

In order that the learner may more readily comprehend the ſenſe of theſe definitions, I have given three different moveable ſchemes; which will alſo be found of great ſervice in illuſtrating the different methods of operation in the following leſſons, by an occular demonſtration.

FIG. 1.

Let the plane A I be raiſed up perpendicular to the plane G D; and the plane S C be brought parallel thereto, ſo as the point S may be perpendicular to, or exactly over the point T. And let A B be an original line anyhow ſituated on the plane G D, and S V a line drawn parallel thereto. Alſo let the thread A *a* S (being drawn up to S) repreſent a ſtraight line drawn from A to S, and S C a perpendicular to the line V A. Then will the plane G D repreſent the Ground Plane, *def.* 2; T the place where the ſpectator ſtands, and S his eye, or the Point of Sight, *def.* 4; A I the Perſpective Plane, or Picture, *def.* 3; and the plane S C the Parallel of the original plane G D, *def.* 7. I N is the Line of Interſection, *def.* 6; V A the Vaniſhing Line of the ground plane, *def.* 8; and C the Centre of that vaniſhing line, *def.* 9. which is, in this caſe, the Centre of the Picture, *def.* 5; and S C the Diſtance of that vaniſhing line, *def.* 9. which is alſo in this caſe the Diſtance of the Picture, *def.* 5. S V is the Parallel of the original line A B, *def.* 7. V the Vaniſhing Point of that line, *def.* 8. and S V the Diſtance of that vaniſhing point, *def.* 9. *s s* are the Seats of the original points A, B, *def.* 10. and *y* the Interſection of the line A B, *def.* 6. The lines *s* C, *s* C, and *y* V, are the Directing Lines, or Directors, of A B, *def.*

def. 11. and the lines A S, B S, are the Viſual Rays of the ſaid line A B, *def.* 12.

FIG. 2.

Bring the plane A I perpendicular to the plane G D, and the point *S* in the plane C *S* Z perpendicular to the point T. Bring down the plane S Z, ſo that the line S Z (which is perpendicular to V A) may coincide with Z *S*. Elevate the plane O M X U, by bringing the plane L *y* M perpendicular to the plane G D, and let the plane O X thereby become parallel to the plane S Z. Alſo let the line S V be parallel to A B, and *S* C perpendicular to V A. Then will the planes G D, A I; the lines I N, V A, *S* C; and the points S, C, *y*, repreſent the ſame as in Fig. 1. The plane S Z V will alſo be the Parallel of the original plane O M X U, *def.* 7. V Z A the Vaniſhing Line of that plane, *def.* 8. and S Z the Diſtance of that vaniſhing line, *def.* 9. The line S V is the Parallel of the original line A B, *def.* 7. V the Vaniſhing point of that original line, *def.* 8. and S V the Diſtance of that vaniſhing point, *def.* 9. The line *y v* is the Primary Director, and *f* V the Secondary Director of the line A B, *def.* 11. Alſo, the points *a*, *b*, are the Ground Seats of the original points A, B, *def.* 11. and therefore the line *a b*, is the Ground Seat of the original line A B.

FIG. 3.

Let the plane A I be raiſed perpendicular to G D as before, and the points S, *S*, *s*, be all brought exactly over the point T, which will then be united in one. Bring likewiſe the plane L *y* M perpendicular to G D, and let the planes V *S V*, V *s v*, thereby become reſpectively parallel to the planes O M X U, L *y* M. Alſo let the line *S* 2 be parallel to the line A B, S C perpendicular to V A, and *S* Z perpendicular to V *V*. In this Scheme as well as the laſt, the planes G D, A I, the lines I N, V A, S C, and the points S, C, *y*, are the ſame as in Fig. 1. The plane V *S V* is the parallel of the original plane O M X U; the plane V *s v* the Parallel of L *y* M, *def.* 7. and therefore the lines *v* V, V *V*, are the reſpective Vaniſhing Lines of thoſe planes, *def.* 8. *S* Z is the Diſtance of the vaniſhing line V *V*, and S *v* the Diſtance of the vaniſhing line V *v*, *def.* 9. The line *S* 2 is the Parallel of the original line A B, *def.* 7; the point 2. the Vaniſhing Point of that original line, *def.* 8; and *S* 2 the Diſtance of that vaniſhing point, *def.* 9. Alſo the line *y* 2 is the Primary Director, *f* 2 the Secondary Director, and *a b* the Ground Seat of the ſaid line A B, *def.* 10, 11.

Note, As a perfect knowledge of theſe definitions is indiſpenſibly neceſſary in the reading of the following leſſons, the ſcholar ſhould not attempt to proceed till he thoroughly underſtands them; for then he will meet with few or no difficulties through the whole book, and the reaſon of every operation will be almoſt ſelf-evident, which, without ſuch *precognita*, might ſeem to him to be involved in obſcurity, and totally inexplicable.

AXIOMS,

Or things which are ſuppoſed to be either previouſly underſtood, or evident in themſelves.

I.

The interſection of two planes is a ſtraight line.

II.

Two parallel lines may be in the ſame plane.

III.

A plane may paſs through a ſtraight line and a point, however ſituated.

IV.

If a ſtraight line interſect two or more parallel lines, they may be all in the ſame plane.

N. B. In every leſſon, I ſuppoſe the Centre and Diſtance of the Picture, and the Dimenſions and Situation of the Object, to be given.

LESSON I.

A Point being given on the Ground Plane, to find its Place or Appearance on the Perſpective Plane; or that Point, where a Line drawn from the Spectator's Eye to the given Point, will cut the Picture.

Operation. By the Seat. From the given point A, draw A *s* perpendicular to the line of interſection I N. And from the ſeat *s*, draw a ſtraight line to the centre of the picture C. Then, from the original point A, to the ſpectator's eye, or point of ſight S, draw the viſual ray A S; the point of interſection *a*, with the director *s* C, is the perſpective of the given point, or the place where it appears on the perſpective plane.

By the Interſection. In any direction draw A P; parallel to which draw S V; and from the point of interſection P, to the vaniſhing point V, draw the ſtraight line P V. Then draw the viſual ray A S, and the point of interſection *a*, with the director P V, is the perſpective of the original point A, as before.

Demonſtration. Let the plane S V C, (Fig. 1.) be brought parallel to the ground plane G D, the point S perpendicular to the point T, and the thread drawn up to the eye at S. Then, becauſe S C is parallel to A *s* (being both perpendicular to the parallel lines V A, I N,) and the plane S V C parallel to the plane G D, it is evident, that a plane will paſs through the lines S C, A *s*, (Ax. II.) and interſect the picture in the ſtraight line C *s*, (Ax. I.) And for the ſame reaſon it appears, that a plane will paſs through the lines S V, A *y*, (A P) and interſect the picture in the line V *y*, (V P). Moreover, it is obvious, that both theſe planes paſs through the points S, *a*, A, which points muſt therefore be in their line of interſection; hence the thread, or viſual ray, paſſing from the original point to the eye, is a ſtraight line, (Ax. I.) And whatever poſition the perſpective plane has in reſpect to the ground plane, if the planes G D, S V C, continue parallel, the viſual ray will always be a ſtraight line, becauſe the points of interſection

S, *a*, A, will manifeſtly continue the ſame: Now let the planes be ſtretched out, and laid down on the ground plane G D; the planes G D, S V, being yet parallel, the thread, or viſual ray A S, will evidently be a ſtraight line, and interſect the lines *s* C, *y* V, (P V) in the ſame point *a*; which is therefore the place where the original point A appears to the ſpectator's eye on the picture.

LESSON II.

A Line being given on the Ground Plane, to find its Appearance on the Perſpective Plane.

Operation. By Seats. From *s*, *s*, the ſeats of the points A and B, (being the extremities of the given line) draw the directing lines *s* C, *s* C; and from the original points A, B, draw the viſual rays A S, B S; which will interſect the directors in the points *a*, *b*. Join *a*, *b*, and it will be the perſpective of the original line A B.

By the Point of Interſection. Produce D E to P, and draw S V parallel to E D; and from the point of interſection P, draw the director P V. And the viſual rays, E S and D S, will interſect the director in the points *e* and *d*. So will *e d* be the perſpective of the line E D.

LESSON III.

To find the Perſpective of any rectilineal Figure.

Operation. By Seats. Find the perſpective of the points A, B, K, by leſſon I. then join the points *a*, *b*, *k*, and it will be the picture of the given triangle A B K.

By the Interſection. Draw S 1 parallel to F H, S 2 parallel to H E, and S 3 parallel to F E. Then from the points of interſection 1, 2, 3, draw the directors 1 1, 2 2, 3 3; and *b e f* will be the perſpective of the triangle H E F.

LESSON IV.

To put a Square in Perſpective.

Operation. Let A B C D be a given ſquare, of which a ſide is parallel to the line of interſection. Having found the ſeats, draw the directors to the centre of the picture, and the viſual rays will interſect them in the points *a b c d*; which being joined will give the perſpective of the ſquare.

If the ſquare be ſituated obliquely to the line of interſection as E F G H, find the points of interſection 1, 1, 2, 2, and the vaniſhing points 1, 2; then the directors being drawn to their reſpective vaniſhing points, will, by their interſections, form the figure *e f g h*; which will be the perſpective of the ſquare when viewed on an angle.

LESSON V.

To find the Perſpective of the Ordinate Figures, a Pentagon and a Dodecagon.

Operation. Let P be the given Pentagon. Find the directing points by producing each ſide to the line of interſection. Then find the vaniſhing points, 1, 2, 3, 4, and draw the directors; which will form the perſpective of the given pentagon by their interſections.

When the figure conſiſts of many ſides, the perſpective is moſt conveniently found by Seats. So to find the picture of the Dodecagon D; find the ſeats of all the angular points; from whence draw the directors to the centre. Then draw the viſual rays, and join the points of interſection; and the figure made thereby will be a true repreſentation of the given Dodecagon.

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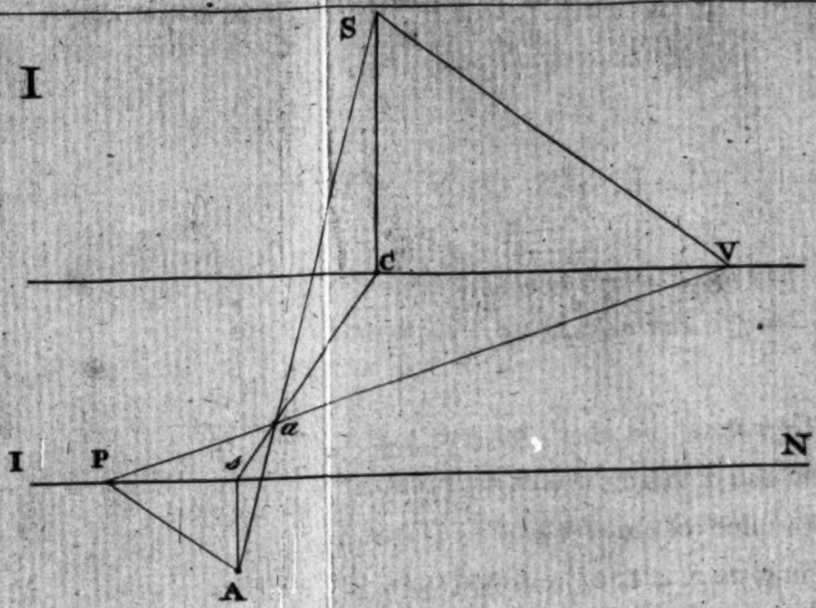

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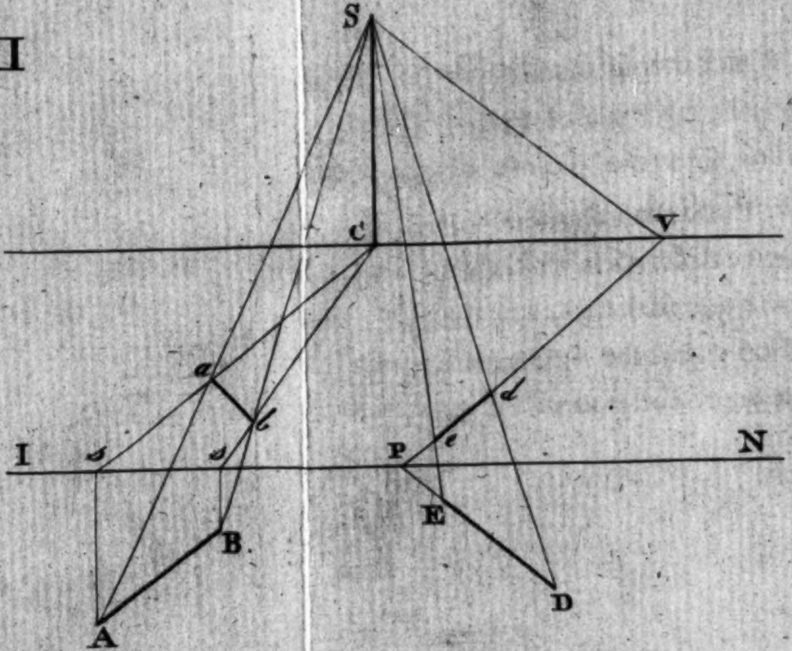

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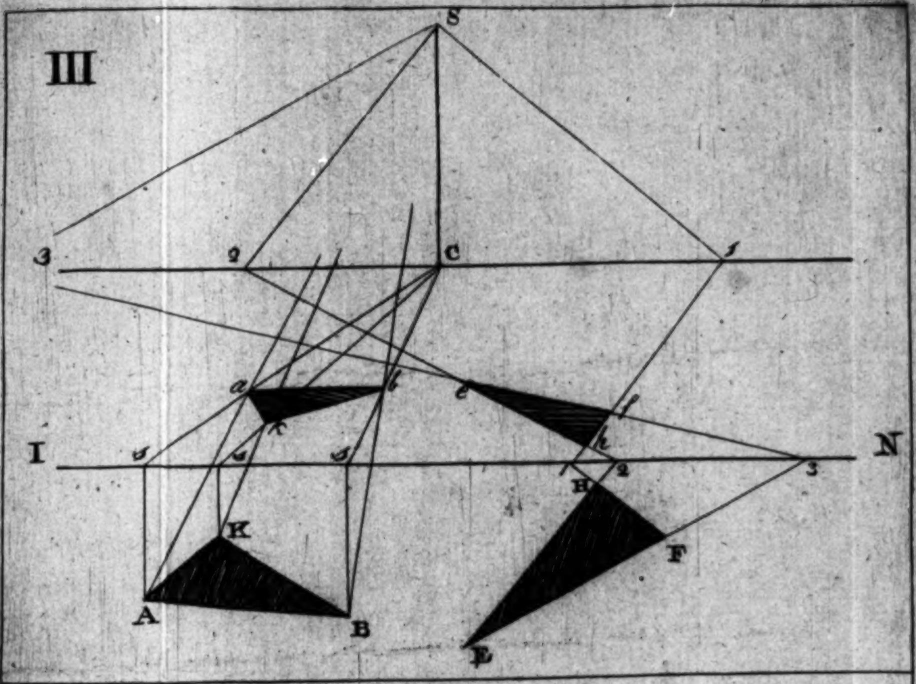

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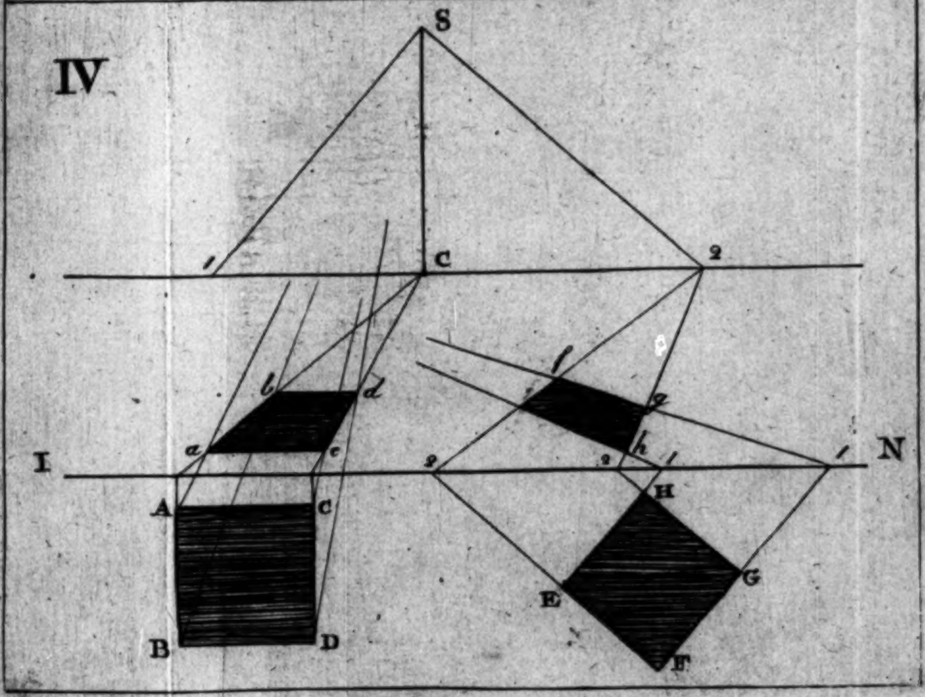


LESSON VI.

To put a Circle in Perſpective.

Operation. *By Seats.* (Fig. 1.) Find the ſeats of any number of points in the periphery; from which draw the directors. Then from the aſſumed original points draw the viſual rays, which will interſect their reſpective directors in the correſponding parts of the periphery; theſe points being joined with a ſteady hand, will ſhew the appearance of the circle on the perſpective plane.

By the Points of Interſection. (Fig. 2.) In any angle find the interſections of any number of points in the periphery, from which draw the directors to their common vaniſhing point V, and they will be interſected in the correſponding points by the viſual rays; and thence the appearance of the circle determined.

It is oftentimes very convenient to circumſcribe the given circle with a ſquare, as in Fig. 3. drawing parallels through the points where the diagonals cut the periphery; and then proceeding after the manner of ſeats.

LESSON VII.

To throw the Ground Plan of an Houſe into Perſpective.

Operation. Find the ſeat of every angle, point, &c. on the ground plane. Then draw the directors and viſual rays, and they will interſect each other in the correſponding points on the perſpective plane. Theſe points being joined, will give the perſpective of the plan.

In regard to the ſemicircle; the perſpective of that may be found by the laſt leſſon, by taking a few points in the periphery.

After the ſame manner may any horizontal plane figure be drawn; as flagged areas, walks, avenues, &c. either of the ſquare, diamond, or groteſque forms.

LESSON VIII.

To find the Appearance of a Pole, of which the Height and Seat on the Ground Plane are given.

Operation. Find the perſpective of the point A as before, and draw indefinitely B E perpendicular to I N, on which take B D equal to the given height of the pole. From the point D draw D C; and from *a* draw *a d* parallel to B D, and it will be the apparent height of the pole.

For a plane being imagined to paſs through B D and the centre C, will paſs through *a d*. And becauſe B C and D C have the ſame vaniſhing point C, they are parallel; therefore *a d* is equal to B D, and perpendicular to the ground plane.

LESSON IX.

To find the Perſpective Appearance of a Parallelogram, which is perpendicular to the Ground Plane.

Operation. Let *A B C D* be the given parallelogram, and A B its ſeat on the ground plane. Find the perſpective of the points A, B, as before; and on the line of elevation take E F equal to *A B*, from whence draw F C (F V); and raiſe the perpendiculars *a c*, *b d*. So will *a c d b* be the appearance of the given parallelogram.

LESSON X.

To find the Perſpective of a Plane Figure ſituated on an Upright Plane.

Operation. Let A B be the ſeat of the given plane, and D E F G the given figure. Produce the ſides to 1, 2, 3, 4; and make E H parallel to A B. Make the angle C *S* V equal to H E 4, and the angle C *S a* equal to H E 2; then will V, and *a*, be the vaniſhing points of the ſides of the figure. On the line of elevation P *g*, take P 1, P 2, &c. equal to *A* 1, A 2, &c. and find the points 1, 2, 3, 4, on the edge of the perpendicular plane; then draw 1 V, 2 V, 3 *a*, 4 *a*, and the interſections *d e f g* will form the perſpective of the figure D E F G.

LESSON XI.

To put a Cube and Parallelopipedon in Perſpective.

Operation. Let the ſquare ABCD be a ſide of the given cube, and *a b c d* the perſpective of it, found by leſſon IV. On the line of elevation take LF equal to AC; and draw the line FC. Draw *a g* perpendicular to *a c*; and make *c b*, parallel and equal to *a g*, and join *g b*. Then to the centre C draw *b e*; and make *d e* parallel to *c b*, and *f e* parallel to *g b*; ſo will the picture of the cube be completed.

Let HIKL repreſent a ſide of the given parallelopipedon. Firſt find the interſecting and vaniſhing points thereof, 1, 2; and the interſections of the directors will form the figure *h i k l*, which is the perſpective of the ſide of the ſolid contiguous to the ground plane. On the line of elevation take 2 *m* equal to IK, and draw the line *m* 2; then raiſe the perpendiculars *i r*, *h p*, and to the vaniſhing point 1, draw *r n*, *p o*. Laſtly, make *k n*, *l o*, each parallel to *i r*, and join *n o*, and there will be formed a true repreſentation of the parallelopipedon. If the line *n o* produced paſſes through the vaniſhing point 2, it ſhew sthat the drawing is accurate.

LESSON XII.

To find the Perſpective of a Square Pillar.

Operation. Let A B C D be the ground plan, or ſeat of the pillar on the ground plane, of which find the perſpective as before. From the ſeat of the line D C, raiſe the line of elevation; on which take L M equal to the given height of the pillar, and draw the line M C. Then raiſe the perpendiculars *d e*, *a f*, and *b g*; and draw *e f* parallel to *d a*, and *f g* to the centre C; and it will produce the perſpective of the given pillar.

If it ſtands obliquely to the line of interſection, find the interſecting and vaniſhing points

1, 2. Then draw the directors; and the interſections will give the appearance of the baſe of the pillar *e f g h*. Make 2 *i* the line of elevation, on which take 2 *i* equal to the given height of the pillar; and draw the line *i* 2, to which raiſe the perpendiculars *e k*, *f l*. From *k* to the vanſhing point 1, draw *k m*, and raiſe the perpendicular *h m*; ſo will the true perſpective of the pillar be had.

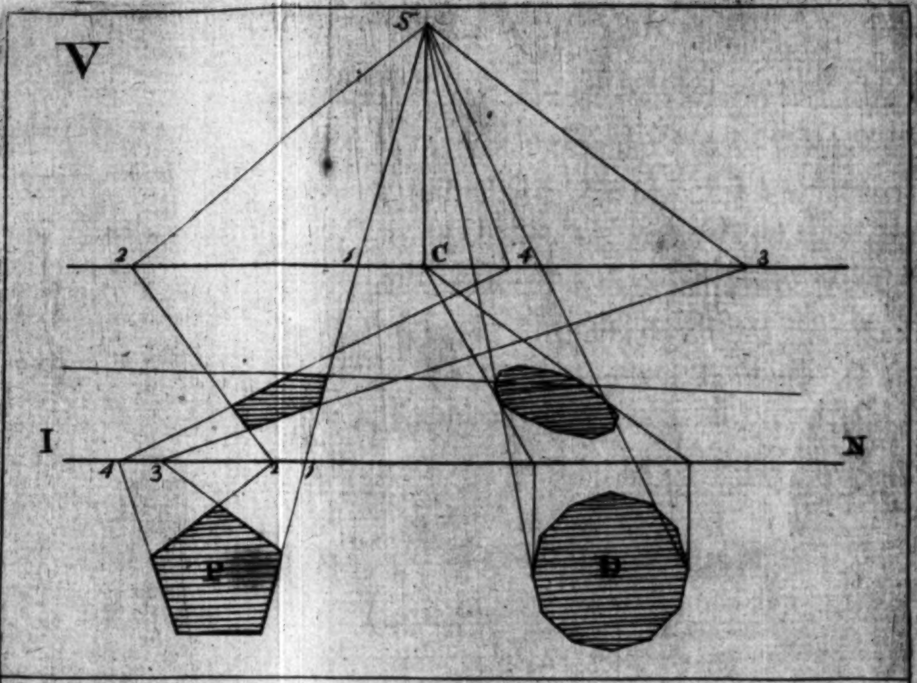
The learner will find little or no difficulty in theſe two laſt leſſons, if he remembers, That all lines which are parallel to one another in the given object, will always have a common vaniſhing point; for from that the reaſon of the operations will be obvious.

LESSON XIII.

To find the Perſpective of a Pyramid.

Operation. Throw the ground plan of the pyramid into perſpective; and then imagine a pole

V



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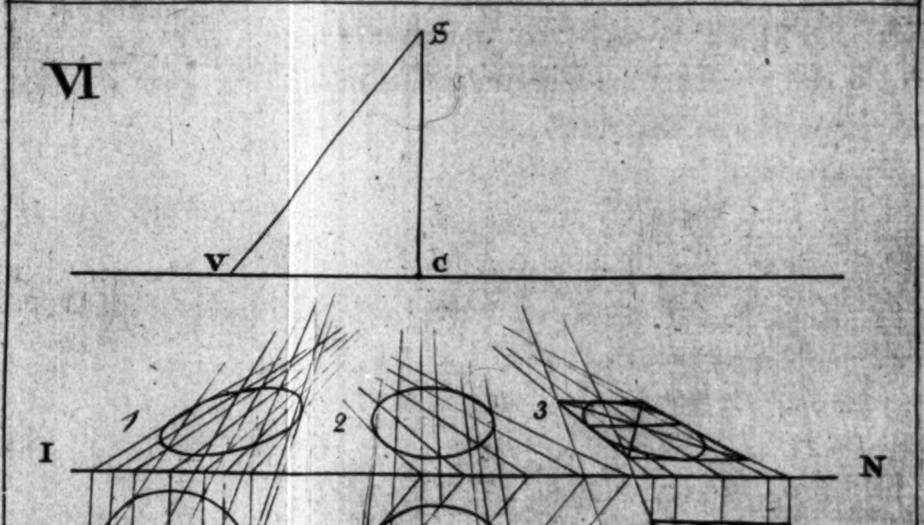

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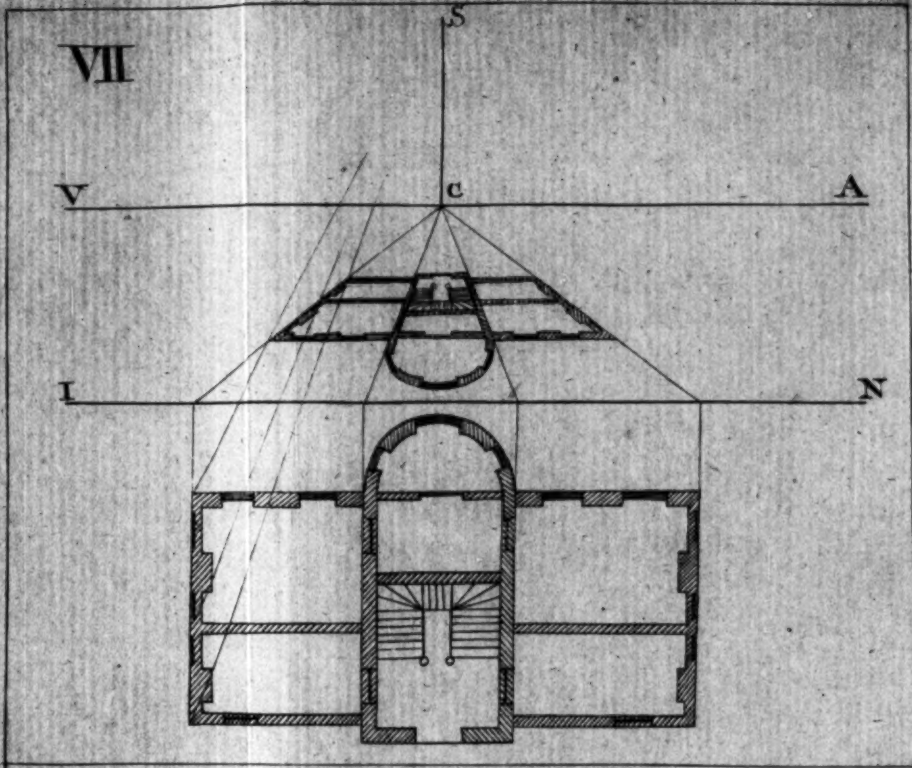

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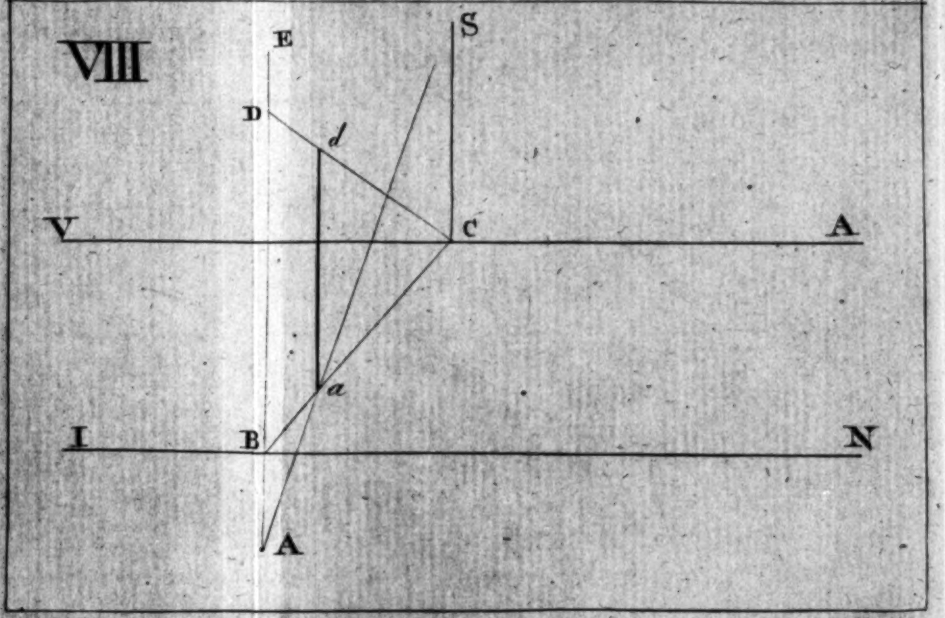

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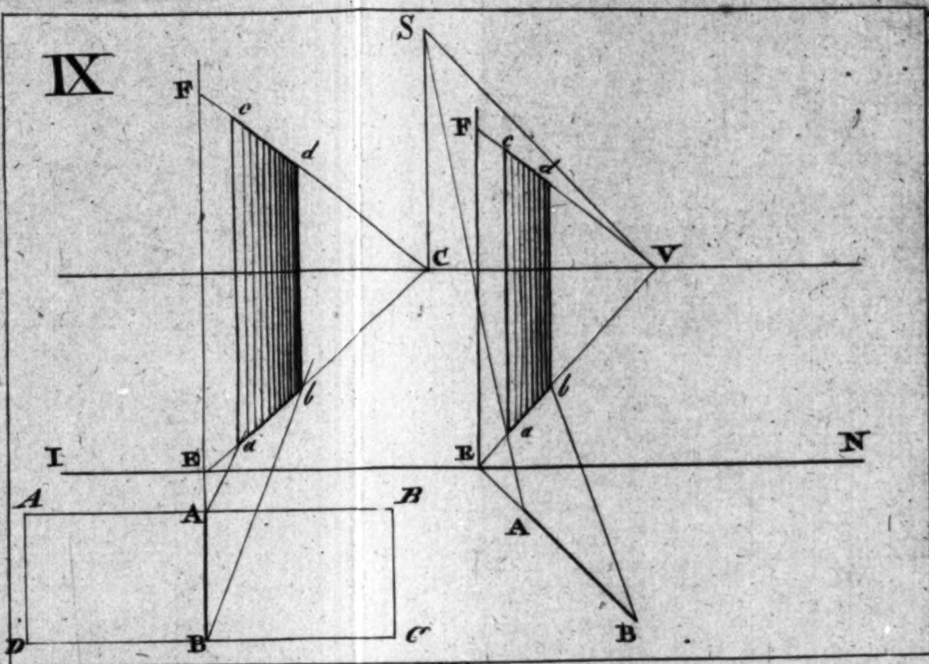

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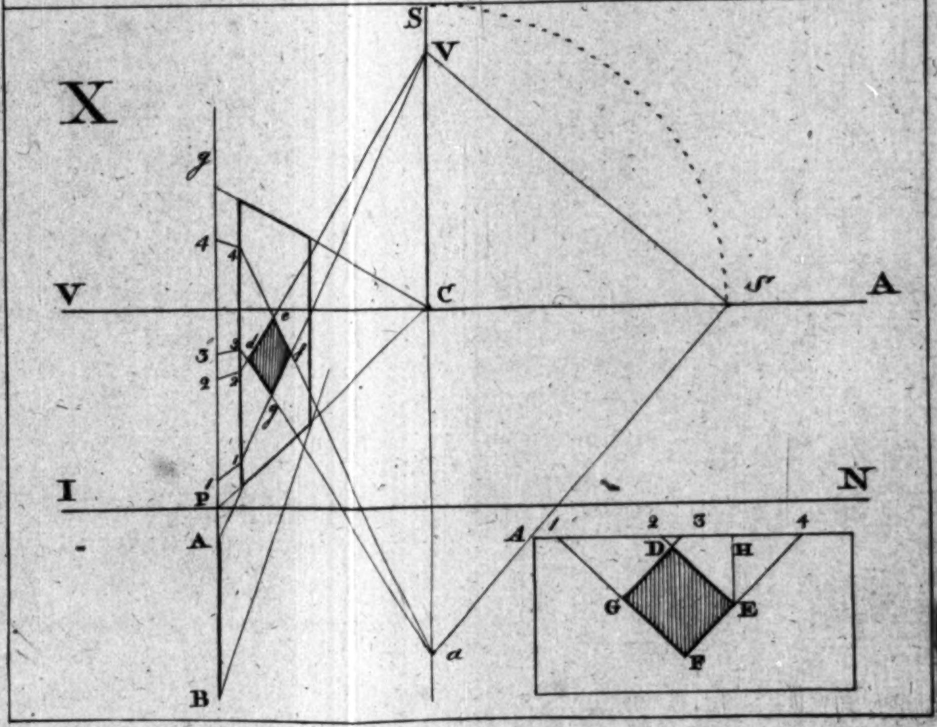

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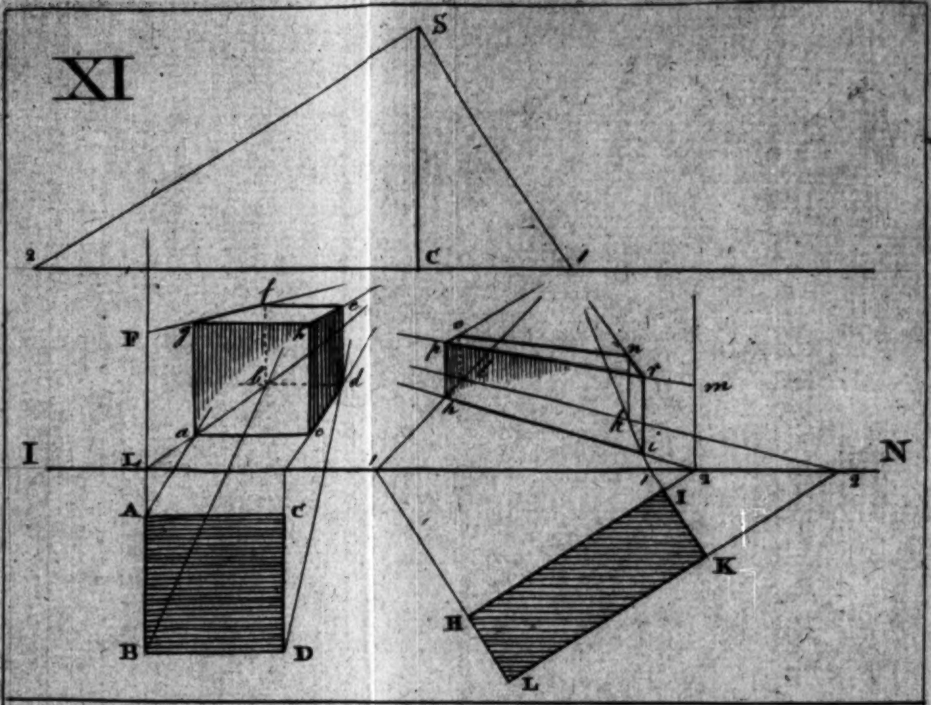
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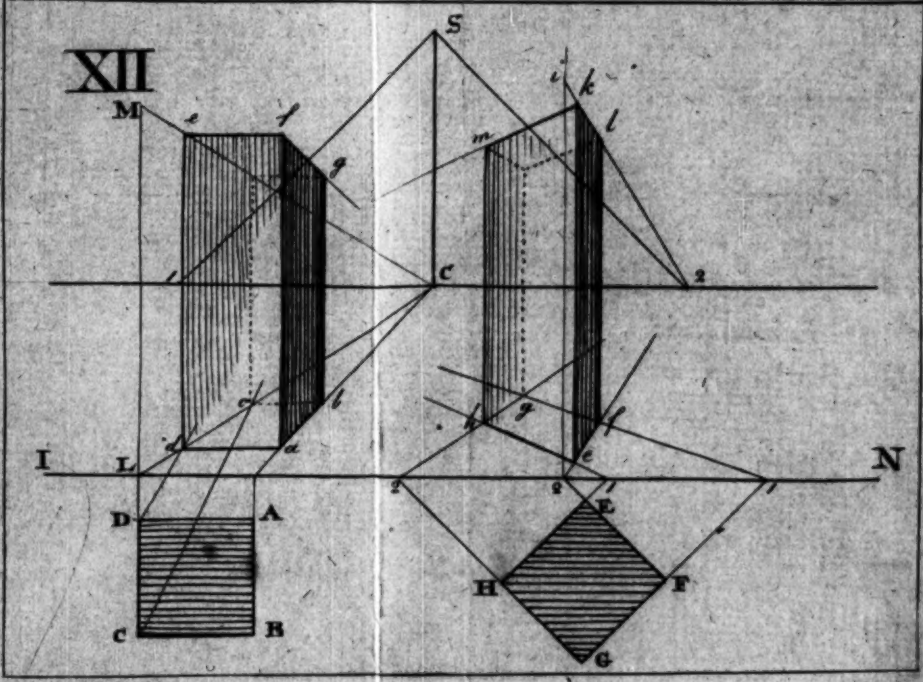
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pole to paſs from the centre of the baſe to the vertex. From the centre of the picture refer the point *e* to the line of interſection, and raiſe the line of elevation L F, which make equal to the given height of the pyramid; then draw F C; and the point where it cuts the perpendicular *e b*, will be the vertex of the pyramid. Laſtly, draw *a b*, *b b*, *c b*, and *d b*; and the perſpective of the pyramid will be completed.

If the pyramid is inverted or ſtands upon its vertex, find the perſpective of the ground plan *e f g*, and the centre *i*, by vaniſhing points, as before. Then make 1 M the line of elevation; and draw M 1, and raiſe the perpendiculars *g g*, *e e*. From *g* draw *g* 2, and raiſe the perpendicular *f f*. Laſtly, join *e i*, *g i*, and *f i*; and the perſpective of the inverted pyramid will be had.

LESSON XIV.

To put an Houſe in Perſpective.

Operation. Let A B D E be the ground plan, and the angle B H D the pitch of the roof. Having thrown the ground plan into perſpective as before, raiſe the perpendiculars *g h*, *f k*; and on the line of elevation, take P F equal to the given height of the raiſing, and *F H* equal to F H. Then draw H C; and make *h k* parallel to *a b*; and *m h*, *n k*, *k o* will be the poſitions of the rafters. The reſt is plain from the figure.

LESSON XV.

To find the Perſpective of an Houſe with an Hip-Roof, viewed upon an Angle.

Operation. Let A B C D be the plan of the raiſing, and F E the height of the pitch. Throw the ground plan into perſpective, and raiſe the indefinite perpendiculars G H, I K, L O. Then, on the line of elevation, take P M equal to the height of the raiſing, and M N equal to F E the pitch of the roof. Draw M *V*, N *V*, and through the interſection H draw the indefinite line *v t*; then raiſe the perpendiculars *d r*, *a t*, draw *t V*, and raiſe the perpendicular *b* X. Laſtly, join *r k*, *t* K, and X O; which will be the perſpective of the Hip-Rafters. The reſt needs no explanation.

LESSON XVI.

To find the Appearance of a Pole inclined to the Horizon; the Angle of Inclination, and the Situation on the Ground Plane being given.

Operation. Let A D be the pole, A B its ſeat on the ground plane G D, and the angle B A D the inclination of the pole to the ground plane. Produce C S, and make the angle C *s* Z equal to the angle B A D: So will Z be the vaniſhing point of the line A D. Find the per-

ſpective

ſpective of the point A by its ſeat *s*; and on the line of elevation take *s g* equal to B D, and draw *g c* and A Z. Then will *a d* be the picture of the inclined pole.

For if we imagine the plane Z *s* C to be brought up perpendicular to Z C, and the plane G D turned back into its proper ſituation, *s* C will be parallel to A B, and *s* Z parallel to A D; becauſe the angles C *s* Z and B A D are equal. And becauſe the points A B D are in the ſame plane, and perpendicular to the ground plane; and *s g* equal to B D, and perpendicular to the line of interſection I N; therefore *s g* and *a d* are in the ſame plane, which is alſo perpendicular to the ground plane G D. Hence *a d* is the appearance of the pole A D on the perſpective plane.

LESSON XVII.

To find the Perſpective of a Parallelogram inclined to the Ground Plane, and parallel to the line of Interſection, having the uſual Data.

Operation. Let O M X U be the parallelogram, O U its interſection with the ground plane, and L O M its angle of inclination. Make the angle C S Z equal to the angle L O M; then will Z be the vaniſhing point of the parallelogram. Having found the perſpective of O U as before, take L M on the line of elevation equal to L M; and draw M C, *o* Z, and *u* Z. Then make *m x* parallel to *o u*, and *o m x u* will be the perſpective of the given parallelogram.

For when Fig. 2. is put in its proper poſition, the plane S Z will be the parallel of the original plane O M X U, (becauſe the angle C S Z is equal to the angle of inclination L O M) and V Z A its vaniſhing line. And ſince S Z is parallel to the ſides of the inclined plane O M and U X, and interſects the perſpective plane in the point Z, it is therefore the vaniſhing point of the ſides of the inclined plane, *def.* 8. And L M is the altitude of the point M above the ground plane, and *o u* the perſpective of O U; therefore *o m x u* is the perſpective of the given plane O M X U.

LESSON XVIII.

To find the Perſpective of a Point, on a Plane inclined to the Horizon, and parallel to the Line of Interſection.

Operation. Let O M X U be the original plane; and *o m x u* the perſpective of it, as before. From B, the given point on the orginal plane, draw B *x* perpendicular to O M, and *x y* perpendicular to O L; then will *x y* be the height of the given point B above the ground plane. Draw B *s* perpendicular to I N, which will in this caſe give the ſeat of the point B, becauſe O U is parallel to I N; then draw the primary director *s* C, and from the point of interſection with the inclined plane, draw the ſecondary director *k* Z. Make *s t* the line of elevation, on which take *s t* equal to *y x*; then draw the line *t* C, and it will interſect the ſecondary director *k* Z in *b*; which is the perſpective of the given point B on the inclined plane.

For if the plane O X (Fig. 2.) be brought to its proper elevation above the ground plane, it will be evident that *x y* is the height of the point B above the ſaid plane. And ſince a plane which is perpendicular to the horizon may be imagined to paſs through B *s*, the point B will be in the interſection of that plane and the original one O X. (Ax. III.) But *k r* is the perſpective of that interſection, and *t s* is the diſtance of the given point from the ground plane; therefore *b* is the perſpective of the point B, as required.

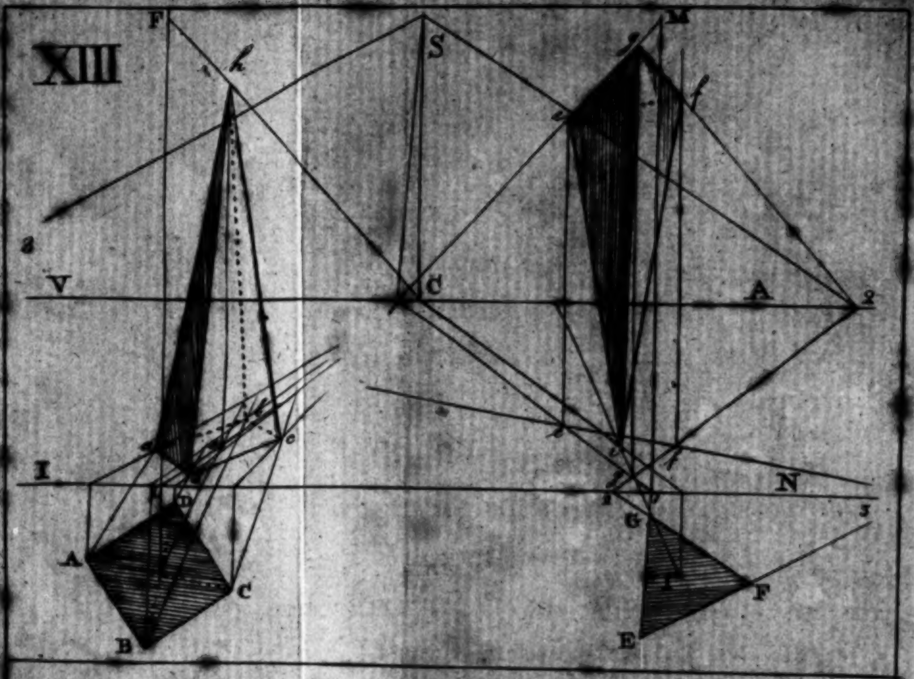
LESSON XIX.

To find the Appearance of a Line ſituated on an Inclined Plane, which is parallel to the Line of Interſection.

Operation. Let *o m x u* be the perſpective of the original plane as before, and A B the given line. Having drawn A X perpendicular to O M, and X Y perpendicular to O L, take O 1 equal to O *y*; and draw 1 *b* parallel to B *x*, and B *b* parallel to O *x*; then will *b* be the ground ſeat of the point B. Produce A B to *c*, and draw *b c y*, and it will give the ground ſeat of a perpendicular plane paſſing through the given line A B on the inclined plane.

Find *v* the vaniſhing point of *b y*, and draw the primary directory *y v*. Take Z S equal to Z *S*, and make the angle Z S V equal to *b* B *c*, and V will be the vaniſhing point of the line A B. Then draw the ſecondary director *f* V, and it will produce an indefinite repreſentation of

XIII



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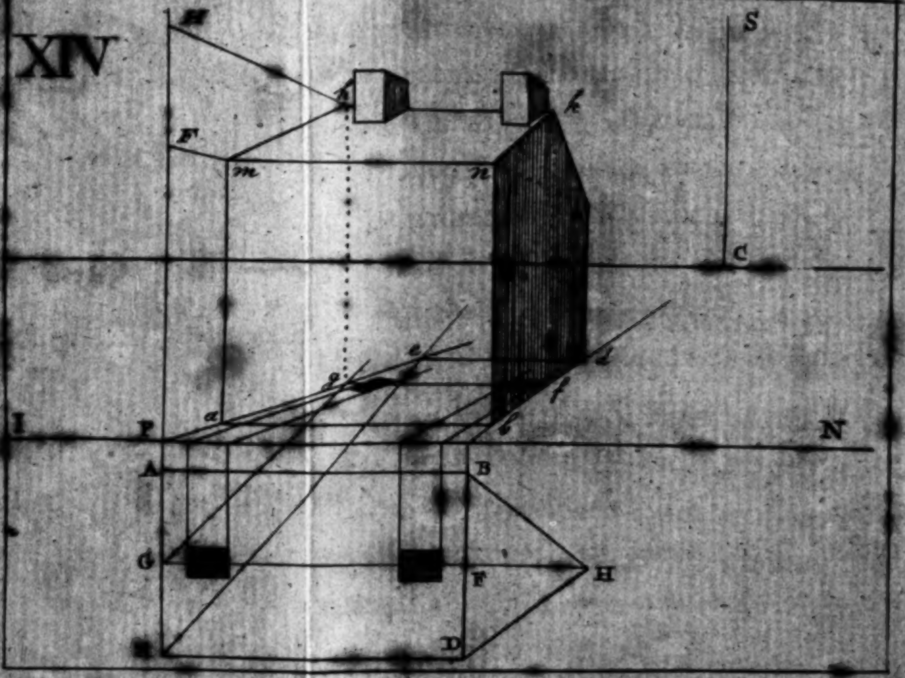

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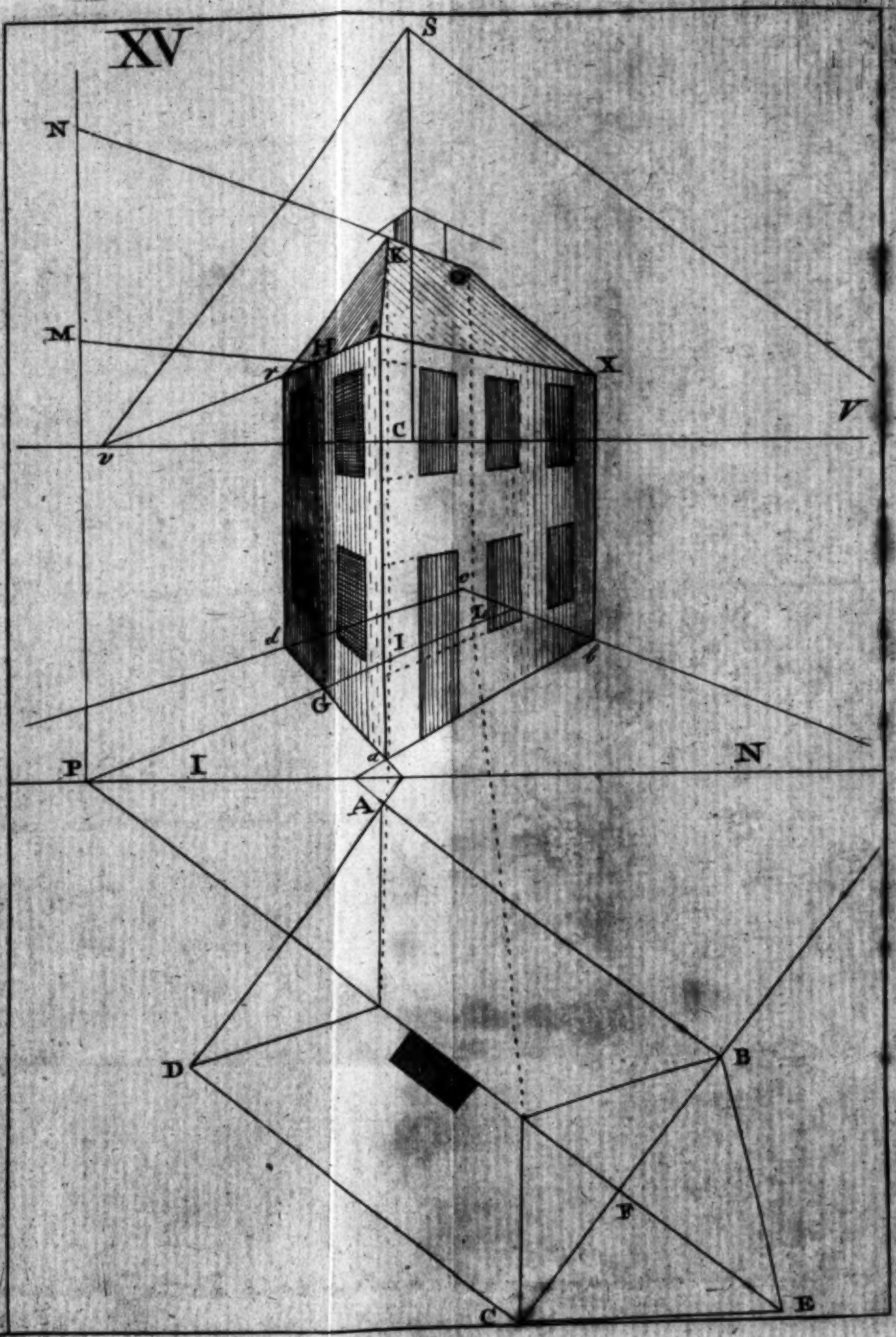

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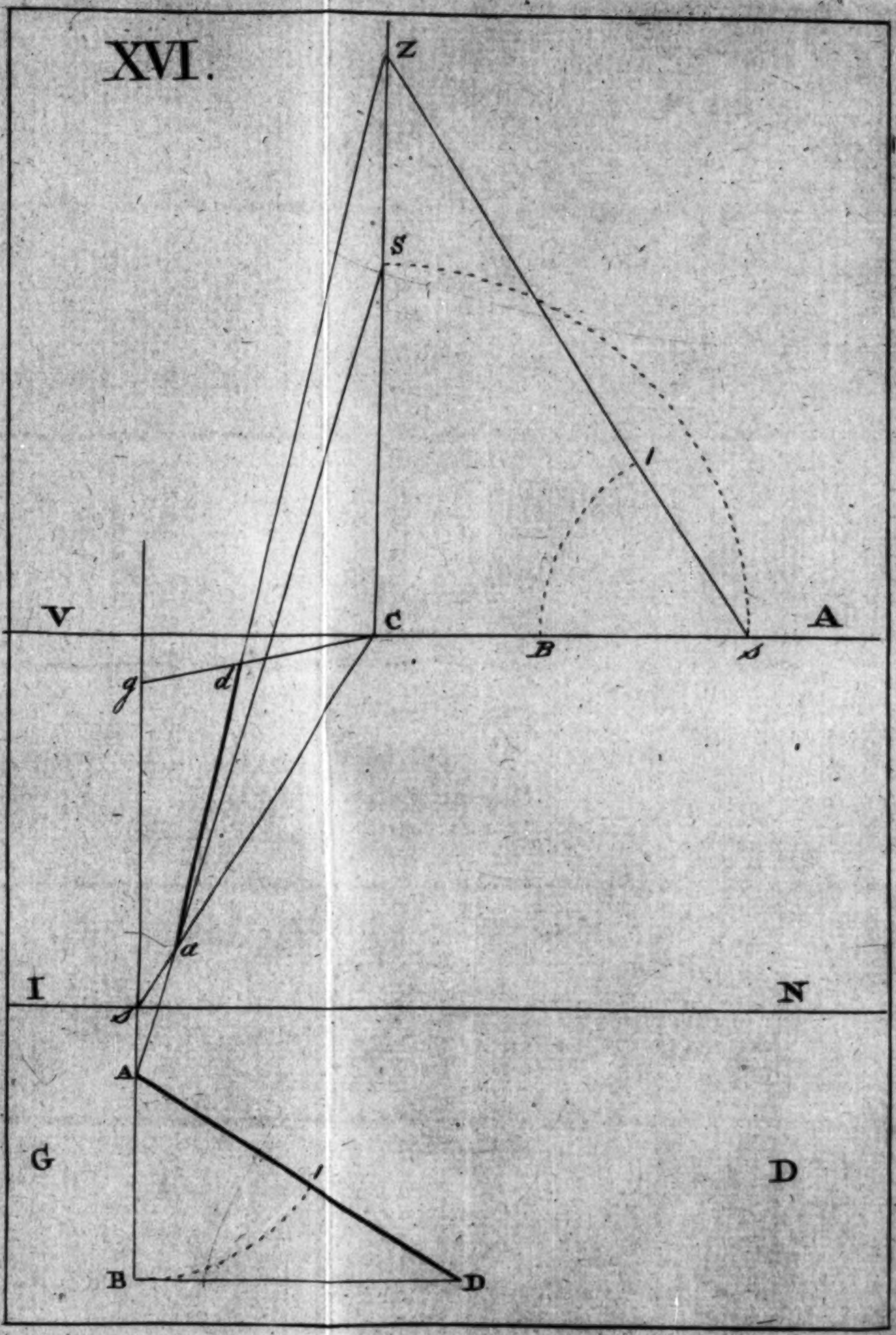


of the line A B. From the point of interſection *y*, make *y* Y a line of elevation, on which take *y x* equal to *y x*, and *y* Y equal to Y X, and draw *x v*, Y *v*; and theſe lines will interſect the ſecondary director in the points *b*, *a*; ſo will *a b* be the perſpective of the line A B.

For let all the planes in Fig. 2. be brought into their proper poſitions, then will *b* evidently appear to be the ground ſeat of the point B; and therefore a plane paſſing through A *c*, perpendicular to the ground plane, will interſect that plane in the line *b y*. (Ax. I.) And becauſe S V is parallel to the line A B, and interſects the picture in V, the line *f* V will be the perſpective of the interſection of the perpendicular and inclined planes. But the line A B is in that interſection, and *y x* and *y* Y are the reſpective heights of the points A and B above *y v*, which is the perſpective of the ground ſeat of the perpendicular plane Y *v y*; therefore *a b* is the repreſentation of the given line A B on the inclined plane.

LESSON XX.

To find the Perſpective of a Figure, ſituated on a Plane inclined to the Horizon, and parallel to the Line of Interſection.

Operation. Let O M X U be the original plane as before, O U the interſection with the ground plane, L O M the angle of inclination, and A B C D the given figure. Make the angle C *S* Z equal to the angle L O M; and parallel to O U draw V Z *A*, which will be the vaniſhing line of the original plane O M X U. Make Z S equal to Z *S*; and draw S V parallel to B A, and S *A* parallel to B D; then will V be the vaniſhing point of the lines A B, C D; and *A* the vaniſhing point of the lines B D, A C. Produce A B to *c*, and C D to *c*. Find *b* the ſeat of B on the ground plane, and draw *b c y*, and parallel thereto draw *c y*; which will be the ſeats of two perpendicular planes paſſing through A B and C D; and therefore *y*, *y*, will be their points of interſection.

Find *v* the vaniſhing point of theſe lines, and draw the primary directors *y v*, *y v*; and from their points of interſection with the inclined plane *f*, *f*, draw the ſecondary directors *f* V, *f* V; which will give an indefinite repreſentation of the lines A B and C D. Produce D B to *p*, and C A to *q*; and draw *p p*, *q q*, each parallel to O L. On the line of elevation take L *p* equal to L *p*, and L *q* equal to L *q*, and and draw *p* C, *q* C; then from the points of their interſection with the ſide of the inclined plane, draw *p A*, *q A*; and *a b c d* will be the perſpective of the figure A B C D on the inclined plane.

For when the planes V N and M L *y* (Fig. 2.) are ſet perpendicular to the ground plane, S *A* will appear to be parallel to B *p*; and therefore the point *A* will be the vaniſhing point of the ſides of the figure B D and A C. And ſince L *p*, L *q*, are the reſpective heights of the points *p*, *q*, above the ground plane, the lines *p A*, *q A*, will be indefinite repreſentations of the ſides of the figure, B D and A C; but by the laſt leſſon the lines *f* V, *f* V, are alſo indefinite repreſentations of the ſides A B, C D; therefore the interſections of theſe lines will form the perſpective of the given figure A B C D.

LESSON XXI.

To find the Appearance of a Pole inclined to the Horizon, and ſituated obliquely to the Line of Interſection.

Operation. Let O M be the pole, O L its ſeat on the ground plane, and L O M its angle of inclination. From *v* the vaniſhing point of O L, take *v s* equal to *v* S; and make the angle *v s* V equal to the angle L O M; then will V be the vaniſhing point of the pole O M. Draw *o* V, and on the line of elevation take *y* M equal to L M, and draw M *v*; ſo will *o m* be the perſpective of the inclined pole, as required.

For let the inclined plane O M X U, (Fig. 3.) be raiſed to its proper angle, and ſuppoſe the ſide of it M O to be an inclined pole. Then, becauſe S *v* is parallel to O L, and *v s* equal to *v* S, the line *v s* will coincide with *v* S, when brought forward to the eye at S, and the plane *v s* V be parallel to the plane L O M. And ſince the angle *v s* V is equal to the angle L O M, *s* V will be parallel to O M, and interſect the picture in V; which is therefore the vaniſhing point of the pole O M. The other part of the operation is plain from the preceding leſſons.

LESSON XXII.

To find the Perſpective of a Parallelogram inclined to the Horizon, and ſituated obliquely to the Line of Interſection.

Operation. Let O M X U be the original plane, L O M its angle of inclination, and O U its line of interſection with the ground plane. Produce M O to *y*, X U to *y*, and U O to *w*; and find the vaniſhing points *v*, *V*, of the lines U *y*, U *w*. Take *v s* equal to *v* S, and make the angle *v s* V equal to the angle of inclination L O M; and V will be the vaniſhing point of the ſides of the inclined plane O M and U X. From *o* and *u* the reſpective interſections of the primary directors *w V*, *y o v*, and *w V*, *y u v*, draw *o* V, *u* V; and on the line of elevation take *y* M equal to L M; then draw M *v*, and through the interſection *x* draw *V m*, and *o m x u* will be the appearance of the inclined plane.

For when Fig. 3. is put in a proper poſition, *s* V will plainly appear to be parallel to O M, and V *v* will be the vaniſhing line of the plane L O M, becauſe that plane is perpendicular to the ground plane, and the angle *v s* V equal to the angle L O M. Therefore V, *v*, and *V* will be the reſpective vaniſhing points of the lines O M, O L, and O U.

LESSON XXIII.

To find the Appearance of a Point, ſituated on an inclined Plane, which ſtands obliquely to the Line of Interſection.

Operation. Let OMXU be the original plane, and B the given point. Find the perſpective of the inclined plane *o m x u*, as before, and draw B *y* parallel to *O y*; then from the point of interſection *y*, draw the primary director *y v*, and from the point of interſection *r*, draw the ſecondary director *r* V. Make B *x* parallel to OU, and *x y* parallel to LM; then will *x y* be the perpendicular height of the point B above the ground plane. Then on the line of elevation *y* M, take *y x* equal to *x y*, and draw *x v*; and through *g*, the interſection with the ſide of the inclined plane, draw *V b*; and the point of interſection *b*, with the ſecondary director *r* V, will be the perſpective of the given point on the inclined plane.

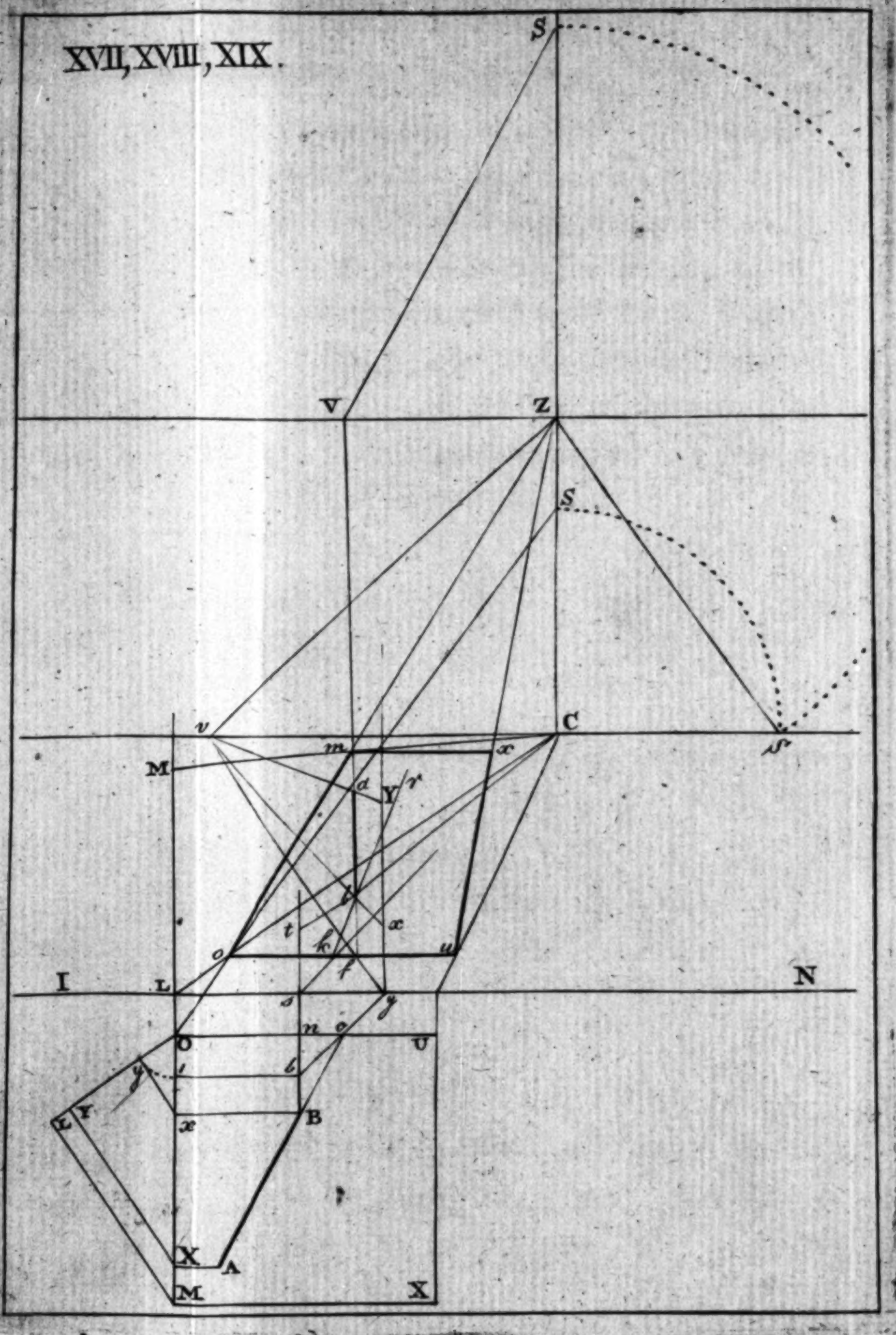
For when the original plane, Fig. 3. is ſet to its proper elevation, *x y* will plainly appear to be the perpendicular height of the point B above the ground plane. And ſince the point B is ſituated in the interſection of the two lines *x* B and *l* B, (which are reſpectively parallel to O U and O M) the perſpective interſection of thoſe lines will evidently be the perſpective of the given point. For *V b* is an indefinite repreſentation of the former, and *r V* of the latter; therefore *b*, their point of interſection, is the perſpective of the given point, as required.

LESSON XXIV.

To find the Perſpective of a Line, which is ſituated on a Plane inclined to the Horizon, and to the Line of Interſection.

Operation. Let A B be the given line on the original plane O M X U. The perſpective of the inclined plane being found as before, find the

XVII, XVIII, XIX.



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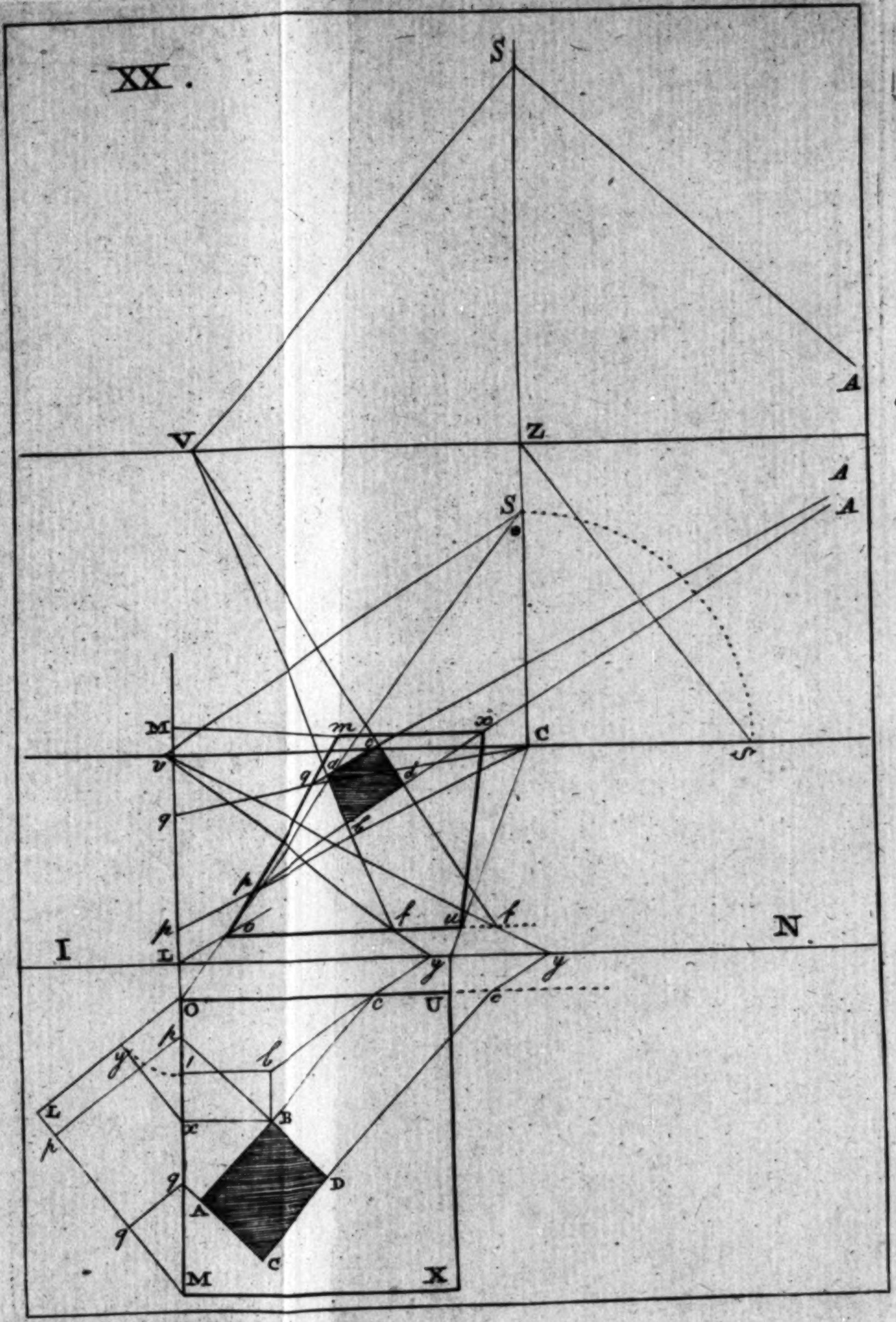

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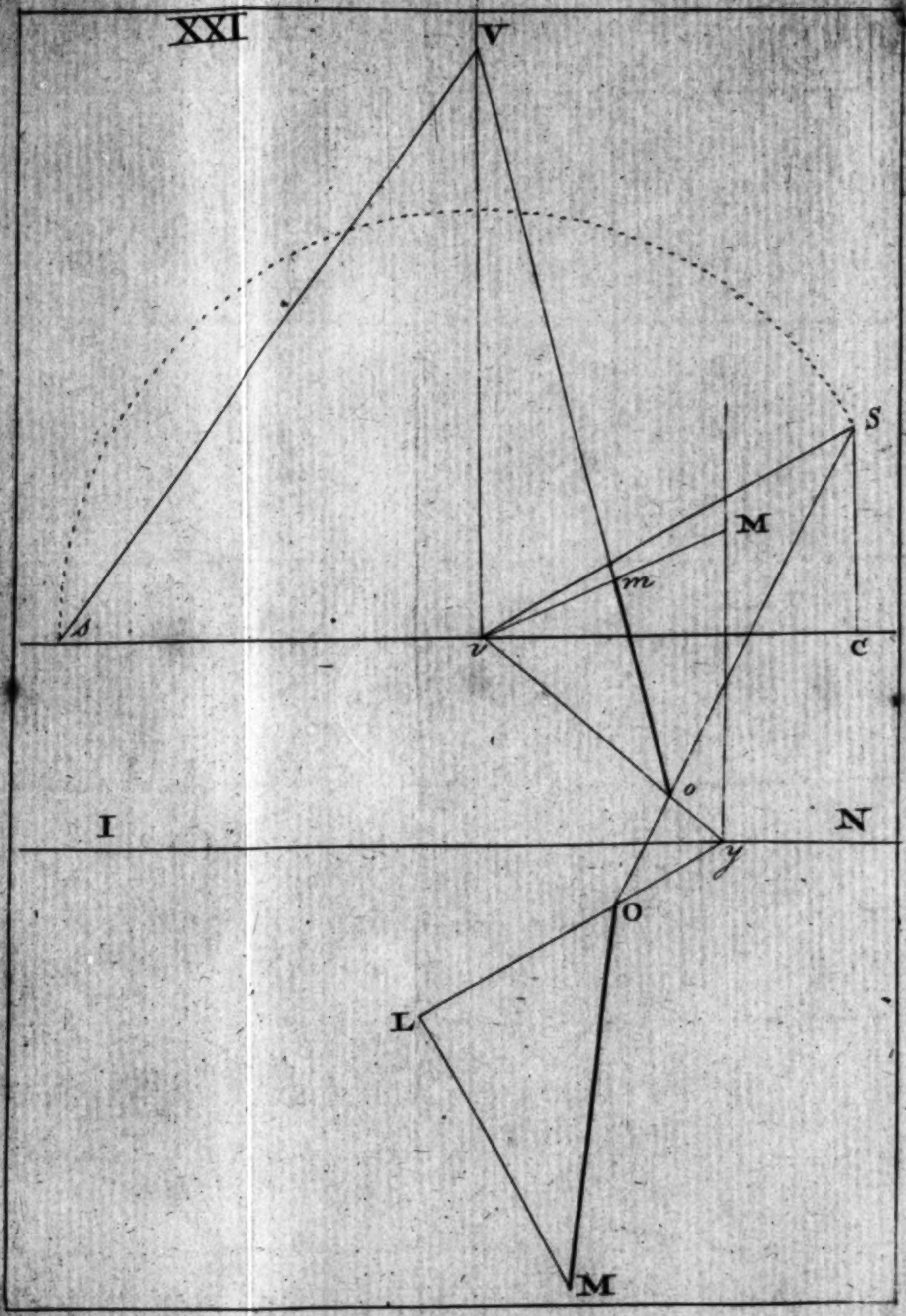


the points of interſection *y*, *y*, of the given points A and B. Draw the primary directors *y r v*, *y t v*; alſo the ſecondary directors *r* V, and *t* V; and on the line of elevation take *y x* equal to *y x*, and *y X* equal to Y X. Then draw the lines *x v*, X *v*; and from the points of interſection with the ſide of the inclined plane draw *V g b*, *V n a*, and join *a*, *b*; and the line *a b* will be the appearance of the line A B on the inclined plane. This is evident from the laſt leſſon.

LESSON XXV.

To find the Appearance of a plane Figure, ſituated on a Plane, inclined to the Horizon, and oblique to the Line of Interſection.

Operation. The inclined plane being thrown into perſpective as before, draw the parallels B *y*, A *y*, and C *y*; and the primary directors *y r v*, *y t v*, and *y e v*. Alſo from the points of interſection with the inclined plane, draw the ſecondary directors r V, t V, and e V. Then, on the line of elevation y M, take the perpendicular heights of the points of the triangle above the ground plane, $y\,x$, $Y\,X$, and Y X; and draw the lines $x\,v$, $X\,v$, and X v. Laſtly, through $g\,b\,n$, the points of interſection with the inclined plane, draw $V\,g\,b$, $V\,b\,c$, and $V\,n\,a$. So will $a\,b\,c$ be the perſpective of the triangle on the inclined plane. This is alſo evident from leſſon XXIII.

LESSON XXVI.

To find the Perſpective of the ſame Triangle, by Vaniſhing Points.

Operation. Take O 1 equal to O y; and draw 1 b parallel to O U, and B l parallel to O M; then will b be the ground ſeat of the point B. And after the ſame manner will c, the ground ſeat of the point C, be found. Produce the ſides of the triangle to p, z, and d; and from the points b and c, draw the lines $b\,p$ 1, *b z* 2, *c d* 3; and they will repreſent the ground ſeats of three perpendicular planes, paſſing through the ſides of the given triangle on the inclined plane. Then find their reſpective vaniſhing points 1, 2, 3; draw the primary directors 1 1, 2 2, 3 3; and through the points V, *V*, draw a ſtraight line, and it will be the vaniſhing line of the inclined plane. Take *V S* equal to *V* S, and V *S* equal to V *s*; and make the angles 1 *S V*, 2 *S V*, and 3 *S V*, reſpectively equal to the angles C *p* U, A *z* U, and A *d* U. Laſtly, from the points of interſection of the primary directors with the inclined plane, *k*, *i*, and *f*, draw the ſecondary directors *k* 1, *i* 2, *f* 3; and they will form, by their interſections on the inclined plane, the triangle *a b c*; which is the perſpective of the original one A B C.

For when the original plane O M X U (Fig. 3.) is brought to its proper elevation, the point *b* will plainly appear to be the ground ſeat of the original point B, *a* the ground ſeat of A, and the line *b c y* (*b z* 2) the ground ſeat of a perpendicular plane interſecting the original plane in the line A B. And after the ſame manner it will appear that the lines *b* 1, *c* 3, are the ground ſeats of two perpendicular planes, paſſing through the lines B C, A C, reſpectively, which are ſituated on the inclined plane. Alſo when all the planes are brought to the eye, perpendicular to the point T on the ground plane, the line *v s* will evidently coincide with *v* S, the line V *s* with V *S*, and the line *V S* with *V* S. And becauſe S *V* is parallel to O U, and *s* V parallel to O M, the plane V *S V* is parallel to the inclined plane O M X U, and interſects the picture in the line V *V*; which is therefore the vaniſhing line of the original plane O M X U. *def.* 8. Now, becauſe *S V* is parallel to O U, and the angle 2 *S V* equal to the angle A *z* U, the line *S* 2 is parallel to the ſide of the triangle A B. And for the ſame reaſon, the lines *S* 1, *S* 3, are reſpectively parallel to the lines B C, A C; and therefore the points 1, 2, 3, in the vaniſhing line V *V*, are the vaniſhing points of the ſides of the triangle.

LESSON XXVII.

To put an Hexahedron in Perſpective, ſtanding upon an angular Point.

Operation. By the Interſections, or vaniſhing Points. Let H K N Q be a ſide of the given Hexahedron. Take K O equal to the diagonal H N, and complete the parallelogram K O P H; which will then repreſent a vertical ſection of the Hexahedron ſtanding on one of its angular points. Upon the diameter F C (equal to I R) deſcribe the regular hexagon A B C D E F; and find *V* and *v*, the reſpective vaniſhing points of A D and B E. Take *v s* equal to *v* S; and make the angles *v s* V, *V v* W, each equal to the angle K H I; and the angle *V v w* equal to the angle P H R. Then will V and W be the vaniſhing points of thoſe interſections of the ſides of the Hexahedron which are not parallel to the picture; and *w* the vaniſhing point of a diagonal. By the interſection of the line A D, A D, find *a*, *g*, the perſpective of the points A, G; and on the line of elevation, take L M equal to H O, and L *p* equal to I K; draw the lines *p V*, M *V*, and raiſe the perpendiculars *a p*, *g r*; then will *p* repreſent one angular point, and *r* the vertex of the given Hexahedron, on the perſpective plane. Now, ſince the interſections of the ſides of the original object, which are perpendicular to the lines A B, F C, and E D, are parallel to the perſpective plane, they will have no vaniſhing point; therefore, draw indefinitely *p u*, *r t*, and *g n*, each parallel to *s* V, and complete the Hexahedron by drawing the lines *r w*, *n h*, *r h*, *r u*, *p t*, and *p g*, as in the figure.

For if we imagine the planes *v s* V, *V* S *v*, and *V v* W, to be each turned up on the lines *v* V, *V v*, and *V* W reſpectively, ſo that the line *v s* may coincide with *v* S, and *V v* with *V* S, then will *s* V plainly appear to be the parallel of the interſections of the ſides of the Hexahedron *p t*, *u r*, and *n h*; *v* W the parallel of *r h*, *u n*, and *p g*; and *v w* the parallel of the diagonal *r n*.

LESSON

LESSON XXVIII.

To find the Appearance of a Dodecahedron.

Operation. By Seats. Let *a b* be a line aſſumed at pleaſure. Take *a e* equal to half of *a b*, from the centre *e* deſcribe the arch *b f*, and draw the ſtraight line *b f*; then from the centre *a* deſcribe the arch *b d*, and draw *d c* parallel to *f b*. Deſcribe two concentric pentagons, of which the ſides A P, C D, are reſpectively equal to *a b*, *b c*; and complete the ground plan as in the figure. Make *F O*, *F I*, *F K*, *F L*, and *F D*, reſpectively equal to F O, F I, F K, F L, and F D; and raiſe the perpendiculars *F G*, *O N*, *K H*, and *D E*. Then make *I G* equal to I L, and *L E* equal to C D; alſo make *G N* parallel and equal to *L E*, and *E H* parallel and equal to *I G*, and join *N H*; then will *G I L E H N* be a vertical ſection of the Dodecahedron, ſtanding on a ſide. Having found the perſpective of the ground plan, take *de* equal to *DE*, *dg* equal to *FG*, and *dh* equal to *KH*; and draw *e*C, *g*C, and *h*C. Refer all the angular points in the perſpective ground plan to the line *d*C, as *bi*, *ck*, &c. from which raiſe perpendiculars to the line *h*C; then will the interſections of the perpendiculars, drawn from the correſponding points in the perſpective plan and ſection, give the appearance of the angular points of the Dodecahedron, as *bq*, *rq*; *pm*, *om*, &c. The reſt is plain from the figure.

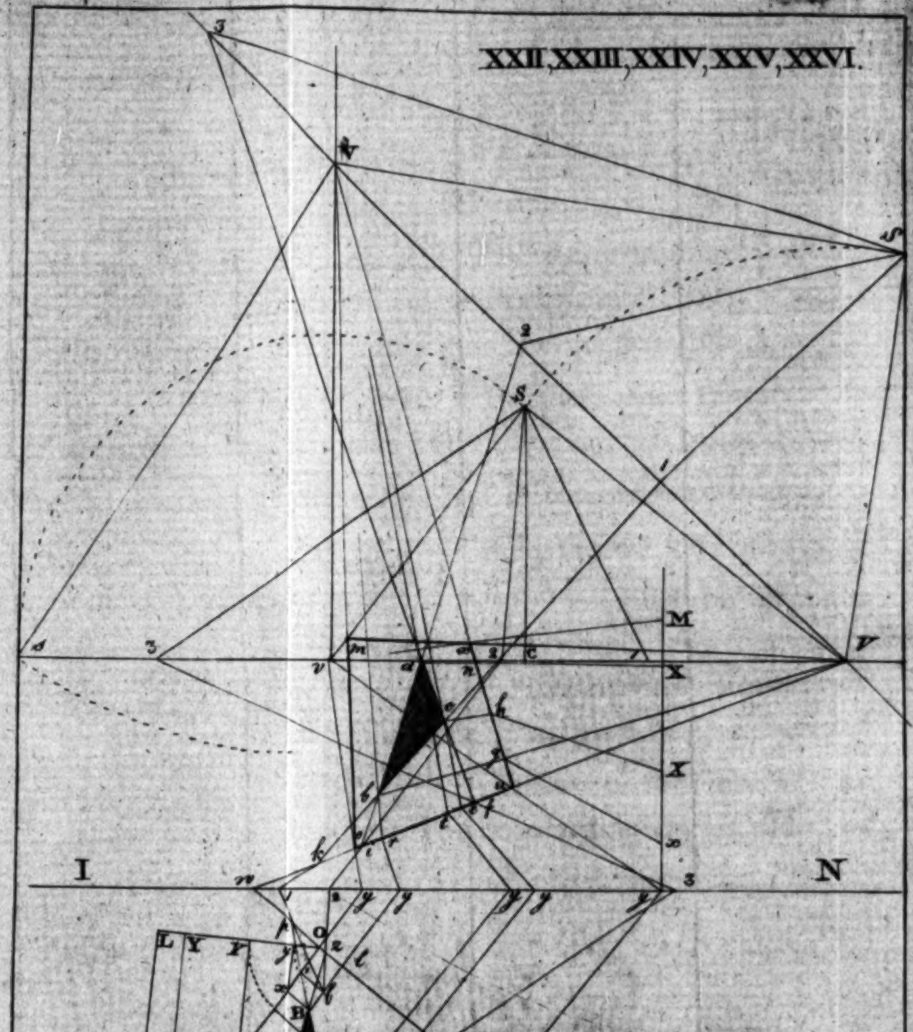
The mathematical reader, I ſuppoſe, will readily perceive from the method of operation, that the ſides of the concentric pentagons are in the ratio of the ſegments of a line cut in extreme and mean proportion.

LESSON XXIX.

To find the Perſpective of an Icoſahedron.

Operation. By vaniſhing Points. Find the lines HE and ID as before; and complete the

XXII, XXIII, XXIV, XXV, XXVI.



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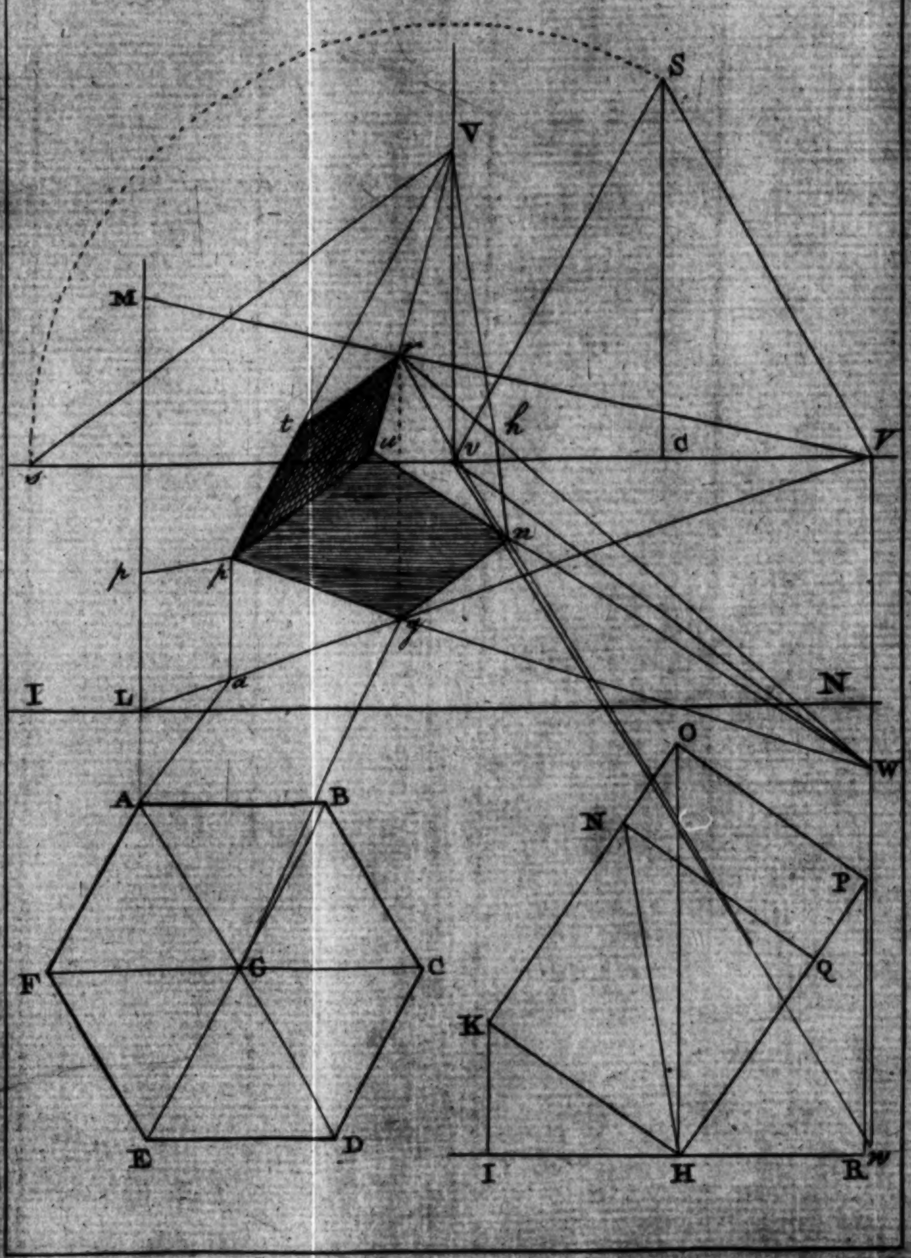

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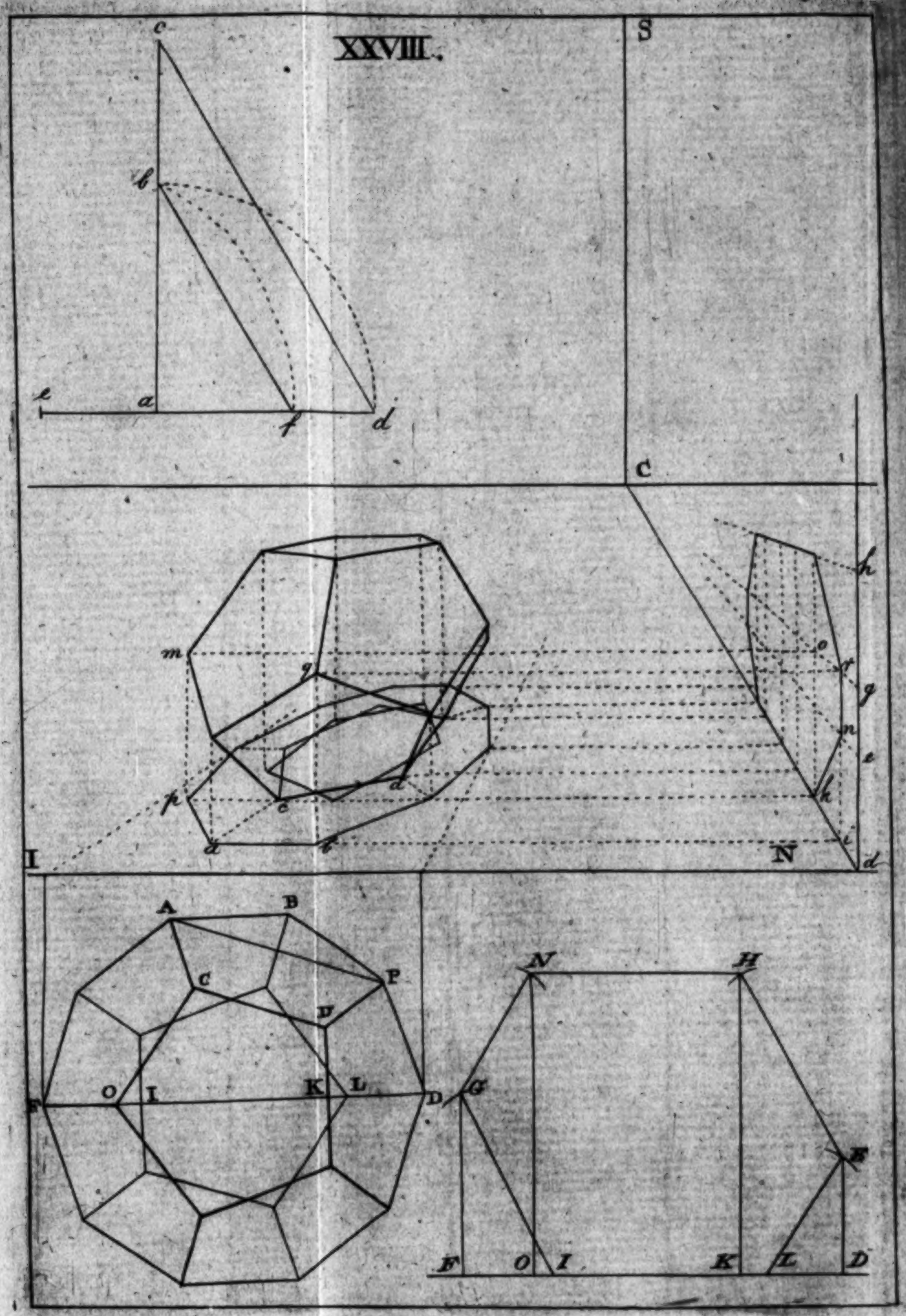

the plan and ſection, as in the figure; then find *S* and *n*, the vaniſhing points of L M and H I. Take *S s* equal to *S* S; and draw *s* V parallel to *A F*, and *s v* parallel to *B E*; alſo take *n y* equal to *S v*, and *n V* equal to *S* V. Then will V be the vaniſhing point of the ſides of the triangular faces H I, P Q; and *v* the vaniſhing point of the perpendiculars of the faces W H, K Q. Alſo *V* and *y* will be the reſpective vaniſhing points of the lines M L, T O, and U L, T Y. And becauſe F A, A B, B E, E D, D C, and C F are all parallel to the perſpective plane, they will therefore be parallel in the picture to their reſpective lines in the ſection. Find *m d* the perſpective of M D, being the ſide of the triangular face M D P, on which the body reſts; *a i o* the perſpective of the horizontal face A I O; and the points *u*, *w*, and *h*. Then from *V*, draw *V m l*; and from *y*, draw *y u l*; alſo draw V *i h*, and *v w h*; and make *d e*, and *b e*, reſpectively parallel to *D E*, and *B E*. Laſtly, join the points *a l*, *i l*, *l h*, *m h*, *h d*, *h e*, *i e*, and *o e*; and the figure formed thereby, will repreſent ſo much of the

Icoſahedron as will be viſible to the ſpectator's eye, at the diſtance C S.

After the ſame manner, and by the ſame vaniſhing points, may the perſpective of the other ſides of the body be obtained; but theſe are here omitted to avoid a confuſion of lines.

LESSON XXX.

To put a Sphere or Globe in Perſpective.

Operation. Let A B Q be the dimenſions of the Globe, and A the ſeat of the centre on the ground plane. Make D R on the line of elevation equal to A B the ſemidiameter of the Globe; from whence find *g* the perſpective of the centre. Make C *S* perpendicular to C R, and equal to C S, and draw *S g*; alſo make R *q* perpendicular to C R, and equal to A D. Take A G equal to *q g*, and G F equal to *g S*; and make the angle F G I equal to the angle C *g S*. Draw I G K, and K L, H I, each perpendicular to A F; and take *g i* equal to G I, and *g k* equal to G K. Take alſo N M, M O, reſpectively equal to K L, H I; and from the centre V deſcribe the ſemicircle O P N, and draw M P perpendicular to N O. Laſtly, biſect *k i* in *d*, through which draw *p m*, perpendicular to *k i*, and equal to P M; then will *k m i p*, to the eye of the ſpectator, be four extreme points on the ſurface of the ſphere; through which an ellipſe being deſcribed, will give the appearance of the whole ſphere on the perſpective plane.

As it would be in vain to attempt to ſhow the reaſon of this operation to any but the mathematical reader, the common learner muſt therefore reſt ſatisfied with the bare operation, and paſs by the following demonſtration.

Imagine the triangle C *S g* to be brought forward, and the triangle R *q g* turned backward, ſo as to be both perpendicular to the perſpective plane. Then, becauſe the lines D R, R C, are both in the ſame plane, R *q* is perpendicular to D R; and ſince R *q* is equal to

A B the diſtance of the centre of the globe from the perſpective plane, and g is the perſpective of the centre, Sg, gq, is a ſtraight line paſſing from the centre of the globe to the ſpectator's eye, and cutting the picture in g. Now it is evident, that the rays proceeding from the extremities of the ſphere to the eye will form a cone; and as this is cut obliquely by the perſpective plane in the angle Sg C, the ſection will be an ellipſe; therefore, ſince A F is equal to Sgq, A G equal to qg, and the angle F G I equal to the angle Sg C, I K will be the tranſverſe of the ſection. And by the property of the curve L K multiplied into I H is equal to the ſquare of the conjugate; but N M is equal to K L, and M O equal to H I; therefore M P is a mean proportional between K L and H I, which is equal to the conjugate, or ſhorteſt apparent diameter of the ſphere. Q. E. D.

Remark. From hence appears the neceſſity of a picture being looked upon from a right point of view, in order to produce a proper effect in the ſpectator's eye. For the figure here drawn is a perfect ellipſe, when placed directly before the eye; but when looked at obliquely at the given diſtance, it appears perfectly circular, and is an exact repreſentation of a globe. And the ſame is to be obſerved in all perſpective drawings whatever; otherwiſe, when the diſtance is taken very near the perſpective plane, or the object ſeen very obliquely, the common rules of perſpective will not exhibit a true repreſentation. But if the eye be brought to the proper point of view, the drawing will no longer appear diſtorted, but will produce the ſame image in the eye as the object itſelf would, if ſeen from a proportionable height and diſtance.

I think we may from hence obſerve, that it would be no bad method, if our capital *landſcape* painters, and even thoſe in *portrait* too, were to write down on the back of the canvas, the height of the Centre, and the Diſtance of the Perſpective plane. For then, the Picture might be placed to the greateſt advantage, ſo that not only the contour, but alſo the light and ſhade in the colouring, would appear to the eye, exactly agreeable to the painter's intention; in which light, it is plain, no drawing can be ſeen from any other point of view.

SCIAGRAPHIC PERSPECTIVE.

Or, the Method of finding the Shadows of Objects, on the Perſpective Plane.

Shadowing from a Radiant Point; as Torch, Candle, &c.

LESSON XXXI.

To find the Shadow of a Pole, which is perpendicular to the Ground Plane; having the Ground Seats, and Heights of the Light and Pole given.

Operation. Let L and P be the reſpective ground ſeats of the light and pole; and I H, N K, their reſpective heights above the ground plane. Having found the perſpective of the light and pole, by the lines of elevation I H, N K;

N K; from *l*, the ſeat of the light on the perſpective plane, draw *l p s*; and from *h*, the point of light, draw *h k s*; then will *p s* be the ſhadow of the pole.

For as the pole is ſuppoſed to be a *line* only, and the light a *point*, a plane may be imagined to paſs through them both (Ax. III.); but this plane will interſect the perſpective ground plane in the line *l s* (Ax. I.); therefore *p s* will evidently be the ſhadow of the pole *p k*, on the picture.

LESSON XXXII.

To find the Shadow of a plane Object, when it is perpendicular to the Ground Plane.

Operation. Let A B C D be the original object, and B C its ſeat on the ground plane; L the original ſeat of the light, and I H the height of it. Produce C B to N, and make N K equal to A B; and from thence find the perſpective of the given object, *a b c d*. From *l*, the perſpective ſeat of the light, through the points *b c*, draw the indefinite lines *l e*, *l f*; and from *b*, draw the rays of light *b a e*, *b d f*; and join *e f*. Then will *b e f c* be the perſpective ſhadow of the object *a b c d*.

For the planes *b l e*, *b l f*, *b e f*, paſs through the ſtraight lines *a b*, *d c*, *a d*, and interſect the ground plane in the lines *b e*, *c f*, and *e f*, reſpectively (Ax. I. III.); therefore the lines *b e*, *c f*, and *e f*, are the reſpective ſhadows of *a b*, *c d*, and *a d*, which are the extremities of the given object; and conſequently *b e f c* is the ſhadow of the whole figure *a b c d*.

LESSON XXXIII.

To find the Shadows of Inclined Planes; ſuppoſe an inverted Pyramid.

Operation. Let A B C be the ſeat of the pyramid on the ground plane; and L the ſeat of the

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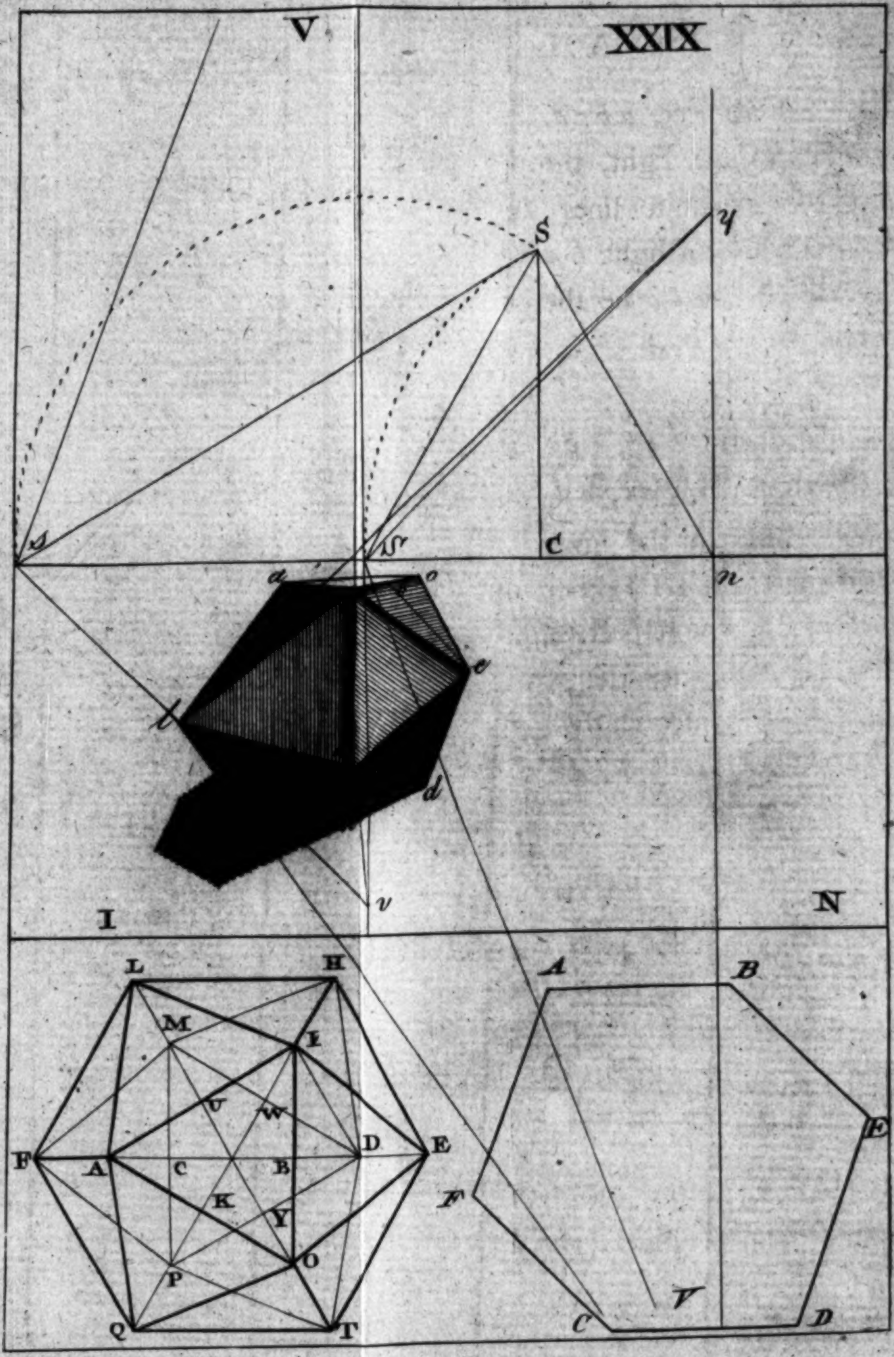

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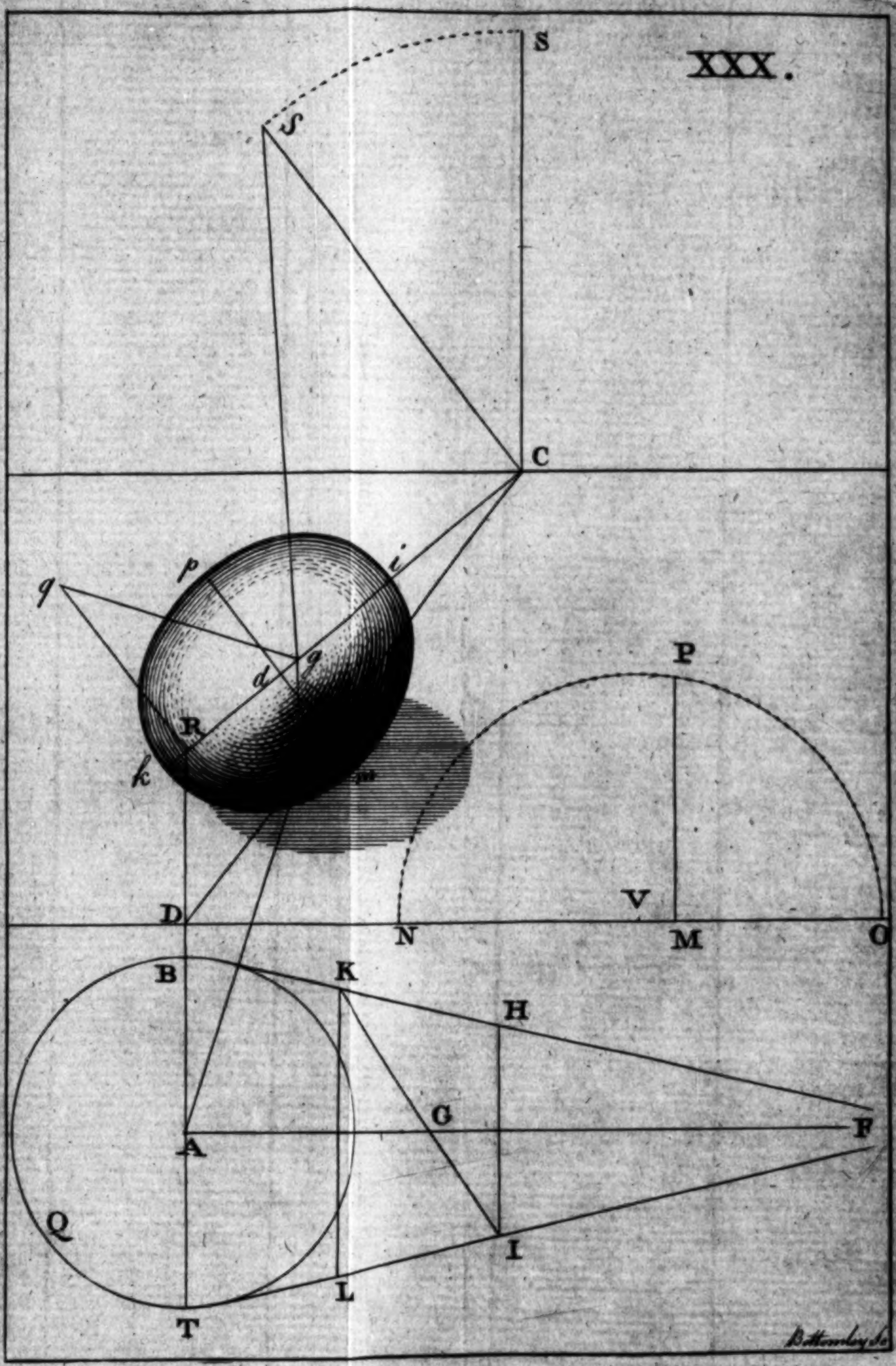

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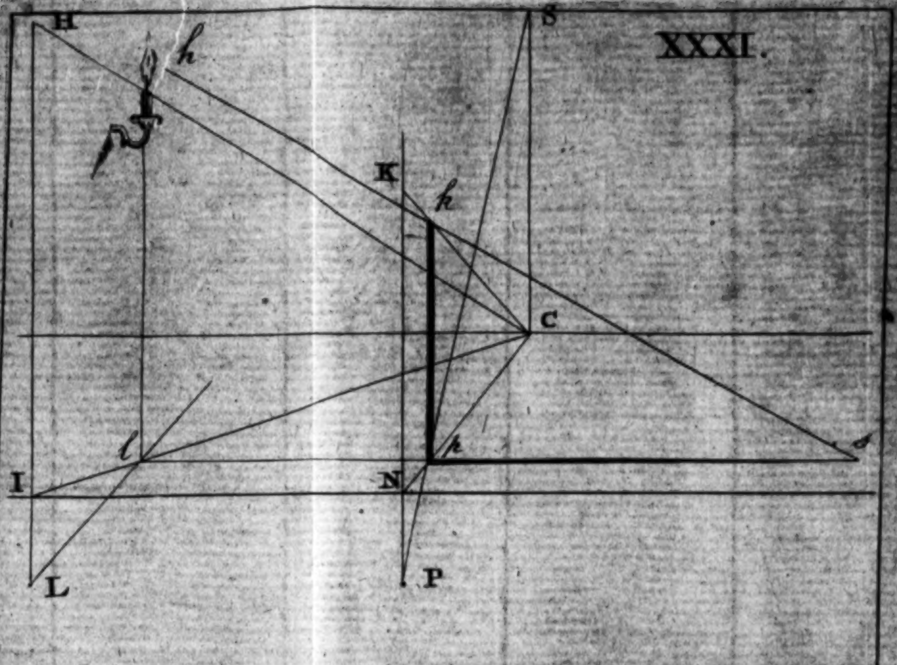

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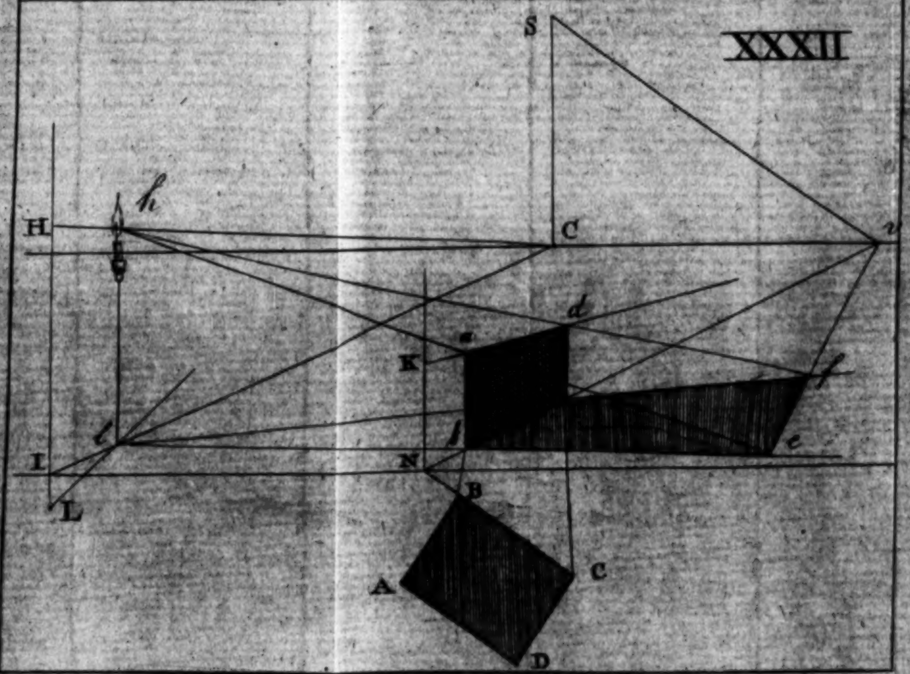

the light. Through L, draw the indefinite line PI perpendicular to the line of interſection; and join AL, BL, CL, and DL; reſpectively parallel to which draw D*a*, D*b*, and D*c*; and they will repreſent the ſeveral interſections of the inclined planes, or ſides of the pyramid produced, with the ground plane. Find *d* the perſpective of D; and upon the director IC, find the perſpective of the points *b*, *c*, *a*; from which draw the indefinite lines *b d b*, *c d c*, and *a d a*. Having found (by the ſeat L and elevation IH) the point of light *b* on the picture, draw the rays *b a a*, *b c c*, *b b b*; and join *b c*, *c a*; then will *d a*, *a c*, *c b*, and *b d*, be the ſeveral perſpective interſections of the inclined planes, which may be imagined to paſs through the extremities of the pyramid, with the ground plane; and therefore the ſpace included *d a c b d* will be the ſhadow of the pyramid.

LESSON XXXIV.

To find the Shadow of a plane circular Ring.

Operation. Let *r p o u* be the appearance of the Ring on the perſpective plane, and *v* the vaniſhing point of its ground ſeat. Draw the lines *vry*, *vpo*, *vum*; and raiſe any number of perpendiculars, as *my*, *ab*, *ur*, &c. Then, from the ſeat of the light *l*, draw the indefinite lines *lmn*, *lag*, *lut*, &c. and the interſections of the rays *bon*, *bzg*, *brt*, &c. will give the correſponding points of the ſhadow; which being joined with a ſteady hand, will form a true repreſentation of the ſhadow of the whole object.

LESSON XXXV.

To find the Shadow of an Hoop.

Operation. Let *bqpr* be the Hoop in perſpective, and C the vaniſhing point of its ground ſeat; *h* the perſpective of the light, and *l* its ſeat on the picture. Through *x*, the centre of the circular edge of the Hoop *b q e r*, draw the perpendicular *r x q*; and from the centre of the picture C, draw the lines C *q a*, C *x b*, C *r c*, and raiſe the perpendiculars, *c a*, *f d*. Then, from the ſeat of the light *l*, draw the indefinite lines *l c i*, *l r t*, *l f s*; and the rays proceeding from the light at *h*, through the points *b*, *q*, *e*, will interſect thoſe lines in the correſponding points of the ſhadow *i*, *t*, *s*; through which and the point *r* the elliptical figure *r i t s r* being traced, will ſhow the ſhadow of the circular edge of the Hoop *b q e r* on the perſpective ground plane. In the ſame manner proceed with the other circular edge *o p m g*, and through the points *m v n u m* deſcribe an ellipſe, and complete the ſhadow as in the figure.

If theſe four points be not thought ſufficient to trace the ellipſe through, we may eaſily find as many more as we pleaſe, by raiſing perpendiculars from the directing line *c* C; and then proceeding as before, by drawing lines from *l* and *h*, through the reſpective ſeats and interſections of the perpendiculars with the edge of the Hoop.

LESSON XXXVI.

To find the Shadow of a Pyramid, when it is intercepted by Objects ſtanding upon the Ground Plane.

Operation. Let the intercepting objects be a parallelopipedon and a wall; which are thrown into perſpective, from the given dimenſions and ſituations on the ground plane. Then imagine a perpendicular plane to paſs through the point of light *b* and the vertex of the Pyramid; and it will evidently interſect the ground plane in the line *l* C; the parallelopipedon in the lines *m m*, *m* 4, 4 *q*; and the wall in the line *o r*. Now it is plain from the figure, ſince the plane *b l* C paſſes through the perpendicular of the Pyramid, that *a* C *b* is the ſhadow of it on the ground plane (ſuppoſing the intercepting objects removed); therefore having produced *m m* to *t*, from the points of interſection 1, 2, draw 1 *t*, 2 *t*; and reſpectively parallel to *a* C, *b* C, draw *m* 4, 2 3. Laſtly from the points of interſection 5, and 6, draw 5 *r* and 6 *r*; and the ſhadow of the Pyramid will be truly repreſented as it falls upon the given objects.

LESSON XXXVII.

To find the Perſpective and Shadow of a Croſs, conſiſting of two plane Parallelograms interſecting each other in any angle.

Operation. Let A B C D be the ſeat of the Croſs on the ground plane, and A E F D its ſection. Find *V* and *v*, the vaniſhing points of B C and A B; take *V S* equal to *V* S; and make the angles *V S* V, *V S* W, reſpectively equal to D A F, and A D E; then will V be the vaniſhing point of the plane A F, and W the vaniſhing point of the plane E D. Find *a b* the perſpective of A B, and raiſe the perpendiculars *a g*, *b b*; then, on the line of elevation, make *l k* equal to A E, draw *k V*, and *v g h*. Having thus determined the points *a*, *b*, *g*, *h*, draw *a* V, *b* V, *g* W, *h* W, and complete the perſpective of the Croſs by means of the vaniſhing point *v*, as in the figure.

From *m* the ſeat of the light, draw the indefinite lines *m b p*, *m a q*, *m r n*, *m o i*; and from the radiant point *h*, draw the rays *h b p*, and *h t n*. Then to the vaniſhing point *v*, draw *p q*, *n i*; and join *a i*, *b n*, *r p*, *o q*, and it will give the perſpective of the ſhadow. For the extremities of the Croſs *g h*, *u t*, are parallel to the ground plane, and *p q*, *n i*, are their reſpective ſhadows on that plane, they will therefore be parallel to each other, and have a common vaniſhing point.

The ſhadow might alſo be determined by drawing one ray of light only, as *h p*; for ſince the points *h t* are in a ſtraight line which is parallel to the ground plane, and having its vaniſhing point in *V*, a line being drawn from *p* to *V* will determine the point *n*; from whence *i* and *q* may be found as before, and thereby the whole ſhadow completed.

Shadowing from the Sun.

The Magnitude of the Earth being but a Point when compared to the Sun's Diſtance, the Rays of Light which fall thereon, will evidently proceed in a Direction parallel to each other. Therefore to find the Shadow of any Object by the Sun, we need only know the Direction of the Rays with reſpect to the Line of Interſection; and their Inclination to the Ground Plane.

LESSON XXXVIII.

To find the Shadow of an upright Plane, when the Sun is in *the Plane of the Picture.*

Operation. Let A B D F be a parallelogram ſeen on the perſpective plane; and *a b c* the angle of the ſun's altitude, or inclination of the rays of light to the ground plane. Draw the indefinite lines A G, and F H, each parallel to the line of interſection I N; and L B G, L D H, each parallel to the ray of light *a b*. Then join G H, and A G H F will be the ſhadow of the parallelogram.

LESSON XXXIX.

To find the Shadow of a perpendicular Plane, when the Sun is behind *the Plane of the Picture.*

Operation. Let A D be the original ſeat of the parallelogram *a b c d*; *p* A the direction of the ſun's rays; and *p* A *r* the angle of the ſun's altitude, or inclination of the rays to the ground plane. Find *V* the vaniſhing point of *p* A, and take *V s* equal to *V* S; then make the angle *V s* V equal to the angle *p* A *r*, and V will be the vaniſhing point of the ſun's rays. Laſtly, from *V* draw the lines *V a e*, *V d f*, indefinitely, and the parallel rays of light V *b e*, V *c f*, will interſect them in the points *f*, *e*; therefore *a e f d* is the perſpective of the ſhadow.

If

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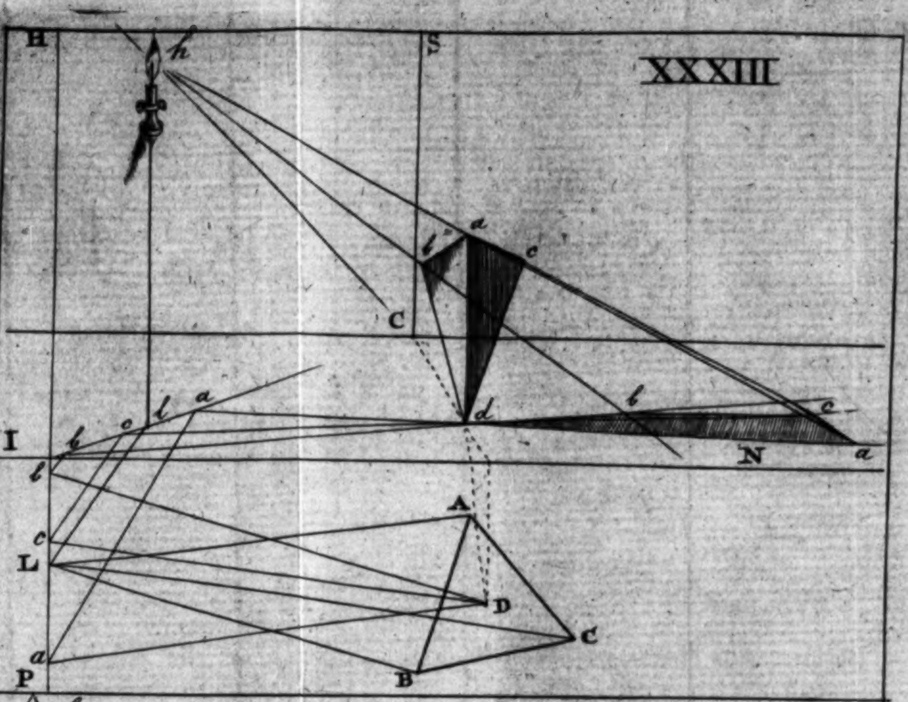

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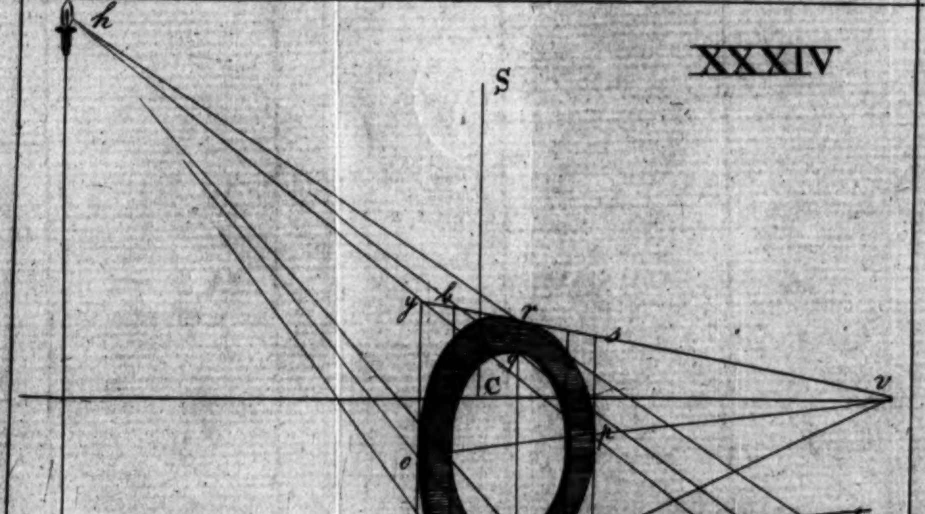

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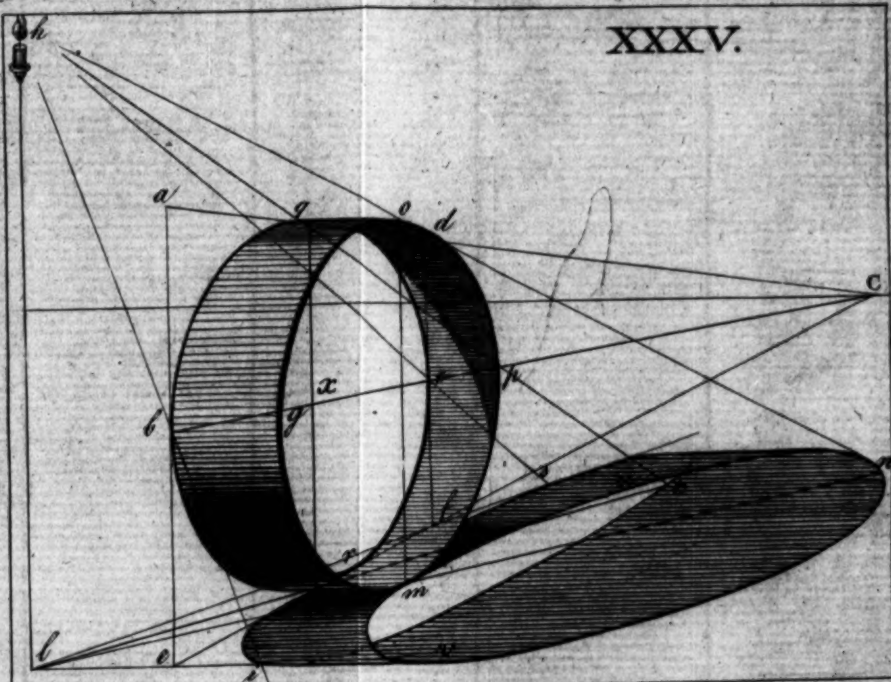

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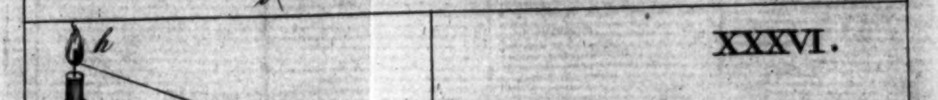

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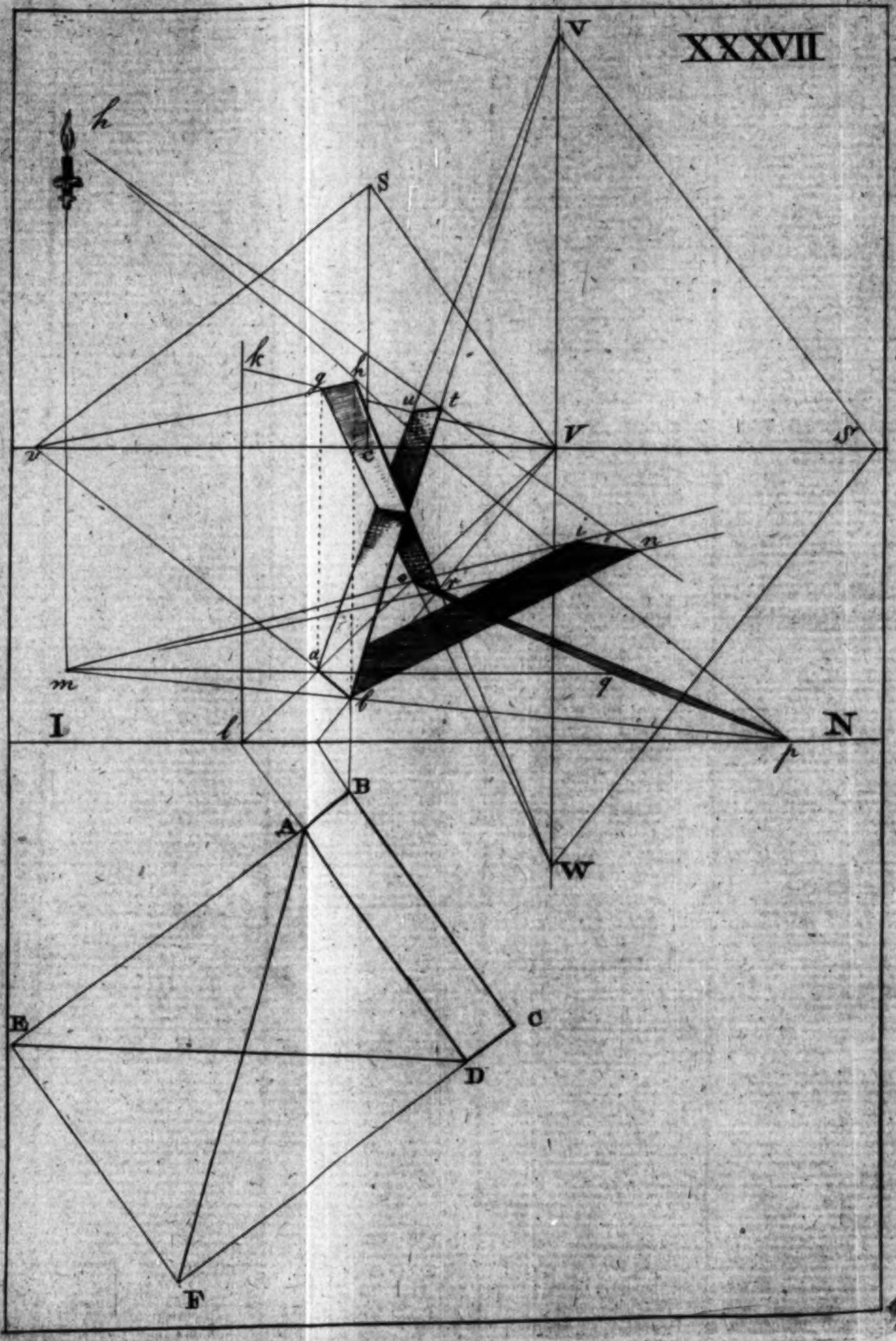

If the line *e f* produced, meets the line of interſection in *W* the vaniſhing point of A D, it ſhows that the drawing is accurate.

LESSON XL.

To find the Shadow of an upright Plane, when the Sun is before *the Perſpective Plane, or Picture.*

Operation. Let A B be the ground ſeat of the parallelogram *a c d b*; *p q* the direction of the ſun's rays; and *r q p* their angle of inclination to the ground plane. Having found *V* the vaniſhing point of *p q*, take *V s* equal to *V* S, and make the angle *V s* V equal to the angle *r q p*; then will V be the vaniſhing point of the ſun's rays. Therefore, by drawing the lines *a V*, *b V*, *c* V, *d* V, and joining the points of interſection *f e*, the perſpective ſhadow of the parallelogram *b e f a* will be determined.

The reaſon of the method of Operation in theſe two laſt leſſons will be obvious, by imagining the plane S C *V* to be turned up perpendicular to the picture, and the plane V *s v* ſo raiſed up that the line *V s* may coincide with *V* S; for then the line *s* V will evidently be parallel to the rays of light, and interſect the picture in V, which is therefore their common vaniſhing point (*def.* 8.) And, moreover, becauſe *V* is the vaniſhing point of the direction of the rays on the ground plane, it is plain that the interſections of the lines, drawn through the extremities of the object to thoſe vaniſhing points, will give the perſpective of the ſhadow of the parallelogram. If this be clearly underſtood, there can no difficulties occur in finding the ſhadow of any kind of object whatever, whether the rays proceed in a direction parallel to each other, or diverge from a radiant point.

CATOPTRIC PERSPECTIVE.

Or the Method of finding the Appearance of an Object on the Perſpective Plane, when reflected from a plane Surface.

An Object ſeen by Reflexion, always appears to the Eye of the Spectator, at the ſame perpendicular Diſtance on the other Side the reflecting Surface, as the Object itſelf is on this Side. Therefore to find the Perſpective of an Object ſeen by Reflexion, we need only ſuppoſe it to be ſimilarly ſituated on the other Side the reflecting Plane; and then proceed according to the foregoing Leſſons.

LESSON XLI.

To find the Perſpective of the Reflexion of an Object, ſituated on an horizontal reflecting Plane, or Surface; as Water, Glaſs, &c.

Operation. Let B R be the brink of the water, and B A a perpendicular pole. Draw B *a* perpendicular to B R, and equal to B A; then will B *a* be the ſhadow, or reflexion of the pole on the water. If the object ſtands at a diſtance from the water (but on the ſame level or plane) as at D, make D *c* perpendicular to B R, and equal to D C; and *c e* will be ſo much of the object as can be ſeen by reflexion in that ſituation of the eye.

To find the ſhadow of the ſquare pillar G F, produce G E, H F, and K I, indefinitely; and make E *g*, F *h*, each equal to the height of the pillar. Then to the centre C draw *h k* C, and *g h k m* will be the reflexion of the pillar on the water. In like manner will the reflexion of the cube M be found, by the vaniſhing points V and *v*; the object and its ſhadow having always a common vaniſhing point.

LESSON XLII.

To find the Reflexion of an Object, when ſituated above the horizontal reflecting Surface; as on a Bank by the Side of a River, &c.

Operation. Let M*n* be the brink of the water, and B*mn*M a perpendicular eminence or bank. Firſt take M*b* equal to MB, and draw the irregular line *bu* for the reflexion of the bank. Then, if the object be cloſe to the edge of the bank as at B, produce AB, and make M*a* equal to MA; then will *ba* be the ſhade, or reflexion on the water. If the object be ſituated at a diſtance from the edge of the bank, as the pole at E; from the centre C draw CEG, let fall the perpendicular GH, and draw H I C; then produce D E indefinitely, and the point of interſection I will be the ſeat of the pole on the horizontal reflecting ſurface continued. Therefore take I *d* equal to I D, and *d i* will be the reflexion on the water. By the ſame method, with the vaniſhing points (V, *v*) of the ſides of the triangular priſm K L *a*, K L *b*, find R, the ſeat of K L; then make R *k* equal to R K, and draw the lines *k v*, *k* V; and the figure formed by the interſections of the perpendicular ſides of the priſm produced, will be the perſpective of the ſhadow.

To find the reflexion of the pyramid, firſt find *o* the ſeat of the centre *q*, as before; then make *o p*, *p x*, *x t*, and *x r*, reſpectively equal to *o p*, *p x*, *x t*, and *x r*; and draw *r p*, *t p*, and it will produce the ſhade or reflexion of the pyramid.

LESSON XLIII.

To find the Reflexion of a Pole from a poliſhed Plane or Looking-Glaſs, when both are perpendicular to the Ground Plane.

Operation. Let A B and P be the reſpective ground ſeats of the looking-glaſs and pole. Draw P *p* perpendicular to A B; and make *r p* equal to *r* P. Having found the repreſentation of the reflecting plane on the picture, *a b f g*; find the perſpective of an upright plane, the ground ſeat of which is P *p*, and height aſſumed at pleaſure. Then will the perſpective of that ſide of the perpendicular plane of which the ſeat is *p*, be the reflexion of the pole on the looking-glaſs.

For the planes *a f g b*, D E *e d*, are perpendicular to the ground plane, and to each other. And becauſe *r p* is equal to *r* P; *d t*, *e n*, are reſpectively equal to *t* D, *n* E, and perpendicular to the reflecting plane; therefore *d e* is the perſpective reflexion of D E.

LESSON XLIV.

To find the Reflexion of a ſquare Pillar viewed on an Angle, when the reflecting Plane is ſituated obliquely to the Line of Interſection.

Operation. Let A B be the original ſeat of the reflecting plane, and D E F G the ſeat of the pillar. Make 1 *d* equal to D 1, 2 *e* equal to E 2, &c. and complete the ſquare *d e f g*; which will then repreſent the baſe of a pillar ſimilarly ſituated on the other ſide the reflecting ſurface A B. Having determined *w*, *x*, the vaniſhing points of E F, E D, the baſe of the given pillar; V, *V*, the vaniſhing points of *d e*, *d g*, the reflexion of the baſe; and *v* the vaniſhing point of D *d*; then find the perſpective of the pillar *m e*, and *n b o r* the perſpective of *d e f g*. Then raiſe the indefinite perpendiculars *t y*, *n o*, *r x*; and to *v* the vaniſhing point of lines perpendicular

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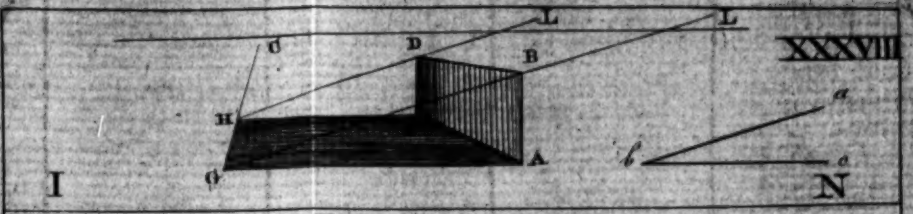

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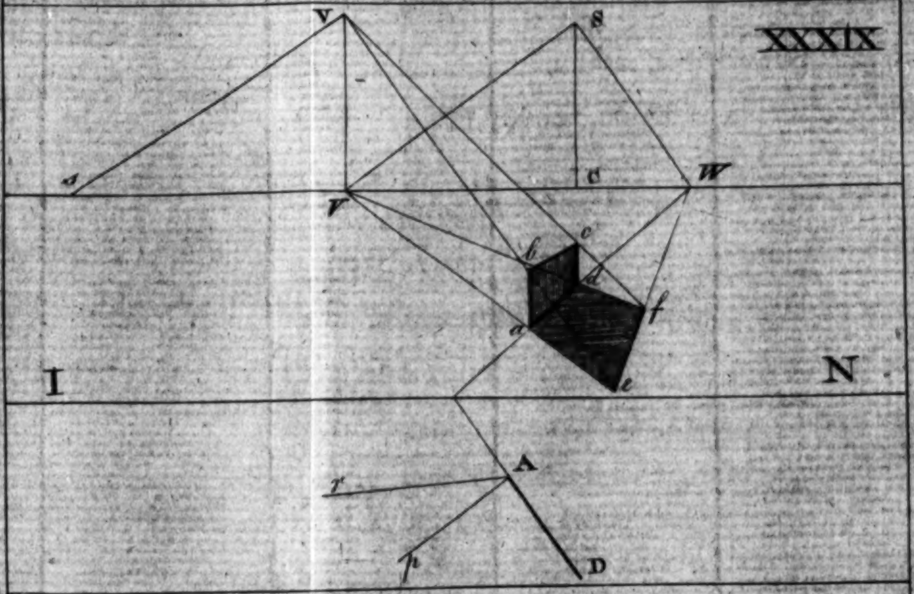

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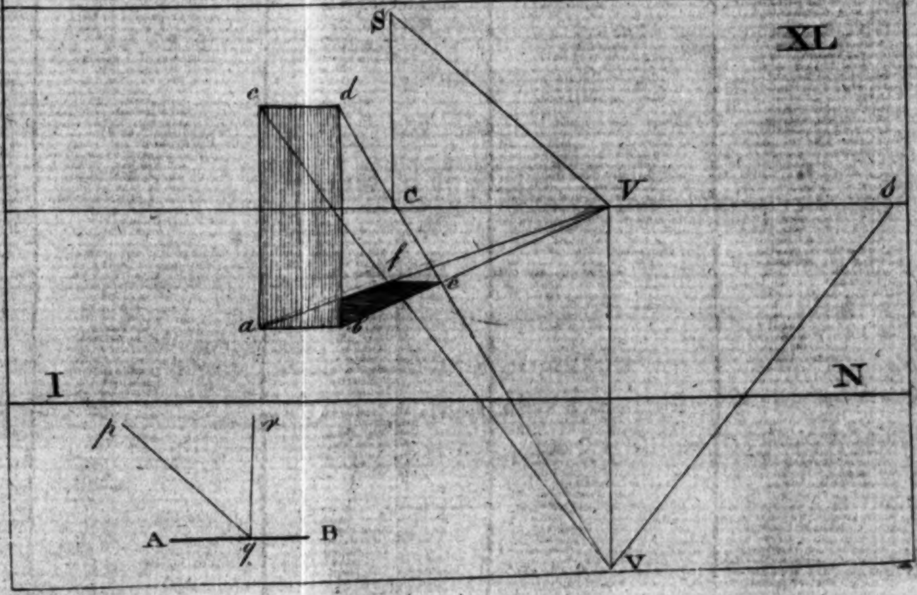

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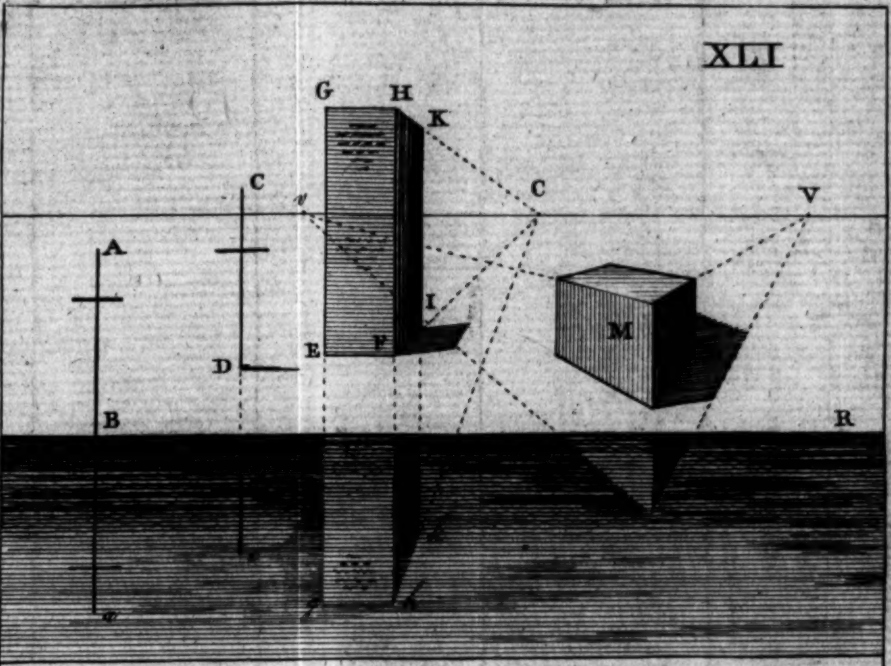

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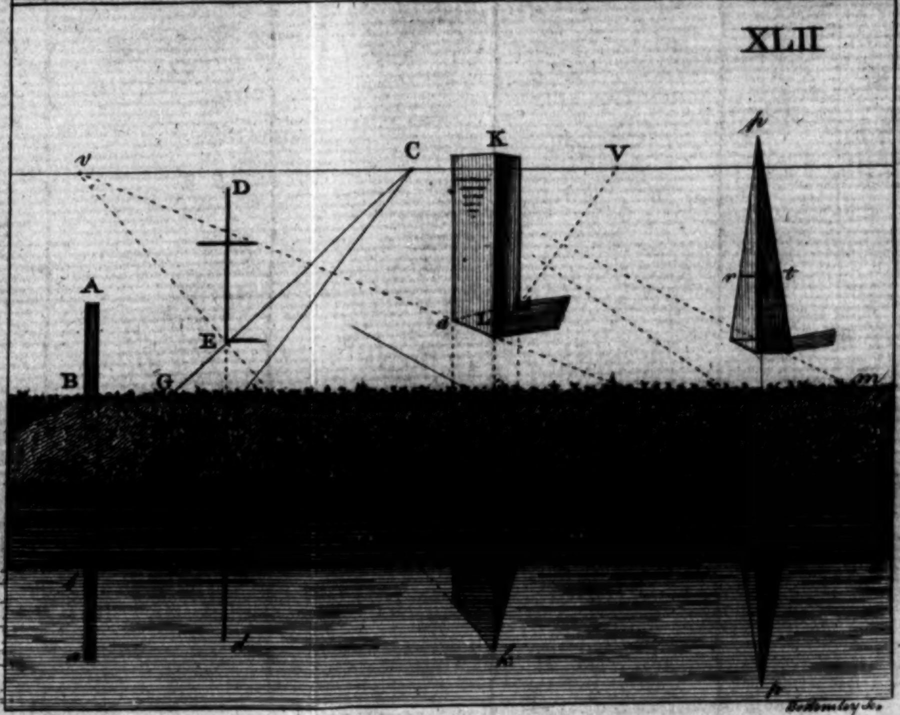

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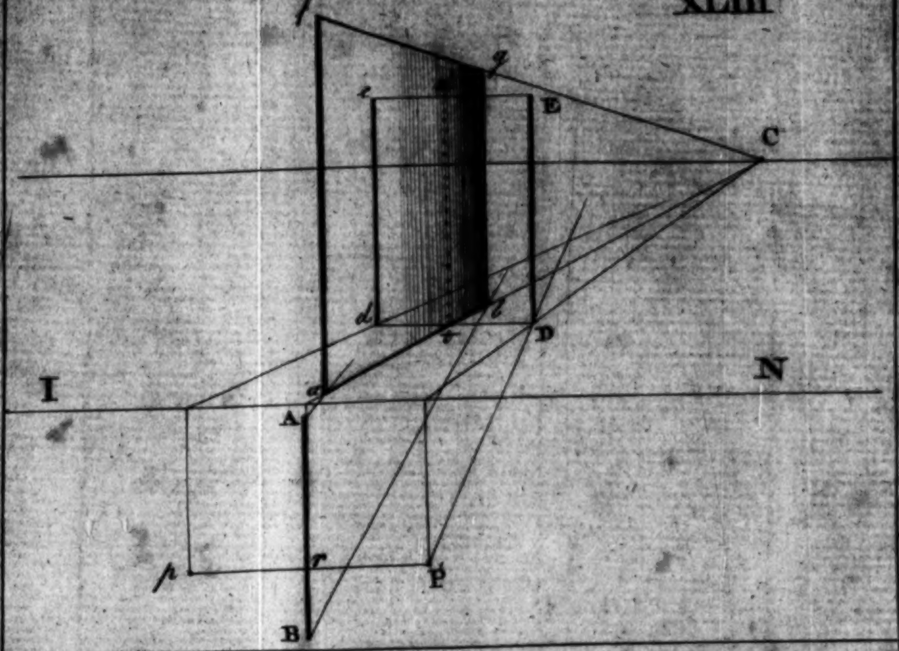

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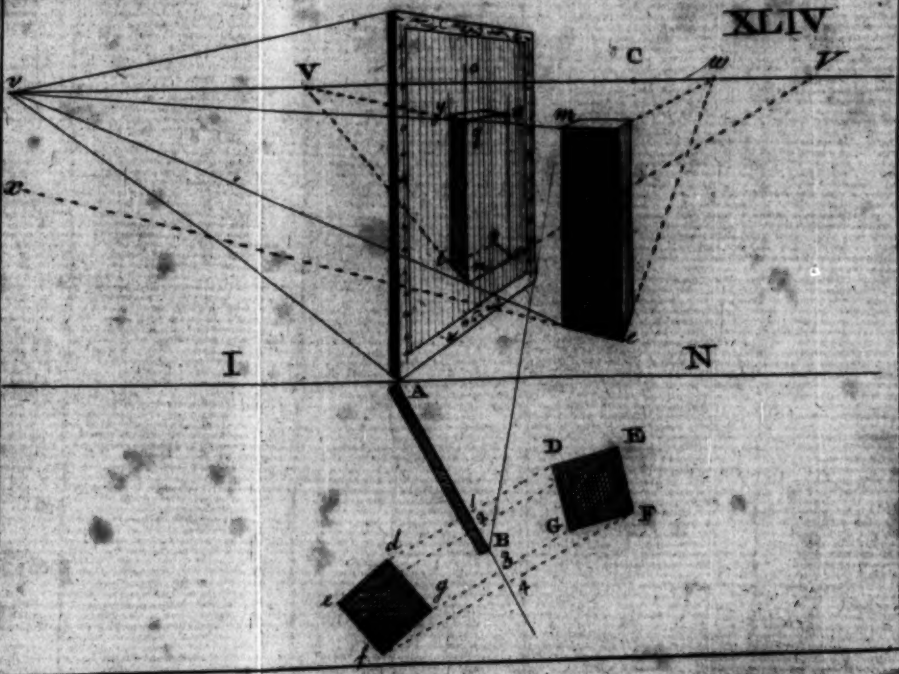


pendicular to the reflecting plane, draw *m v*; and the point of interſection *q* with the perpendicular *n o*, will be the correſponding point in the reflexion; from whence the whole may be completed by the vaniſhing points V *V*, as in the figure. Or, the repreſentation of the reflexion might be ſhown without the vaniſhing point *v*, by finding the perſpective of a pillar of the ſame height with the original one, and ground ſeat *d e f g*.

LESSON XLV.

To find the Perſpective of the Reflexion of a perpendicular Pole, when the reflecting Surface or Glaſs is inclined to the Ground Plane.

Operation. Suppoſe A B to be the ſeat of the glaſs, D B E the angle of inclination, and P the ſeat of the pole. Having thrown the glaſs and pole into perſpective by leſſons VIII. and XXII. from the centre C, draw C H perpendicular to *v* V the vaniſhing line of the reflecting plane, and produce it indefinitely towards W. Take CK equal to CS, and perpendicular to HW; then draw KH and KW, making the angle HKW a right one; ſo will W be the vaniſhing point of lines perpendicular to the plane of the glaſs. Make the angle DBF equal to DBA, and the angle *Vs*Y equal to FBE; then will Y be the vaniſhing point of the reflexion of the pole. Draw OW and TW; and OTW will be a plane perpendicular to the glaſs, and interſecting it in the line *r*N. Laſtly, from N, the point where the pole cuts the reflecting plane continued, draw NY; and *ot* will be the perſpective of the reflexion of the whole pole TO, ſuppoſing the plane of the glaſs to be continued downwards; and *ox* that part of it which will be viſible to the eye at the diſtance CS.

Or, find Z the vaniſhing point of BH, the angle of the reflexion of PB with the ſeat of the glaſs, to which draw *n*Z, and it will cut TW in *t*; from whence draw *t*Y, which will give *to*, as before, for the reflexion of TO.

LESSON

LESSON XLVI.

To find the Appearance of the Reflexion of a Pyramid, from an inclined Looking-Glaſs.

Operation. From the ſeat of the glaſs A B, and angle of inclination to the ground plane D B H, find the vaniſhing line of the glaſs V *v*. From whence find W, the vaniſhing point of lines perpendicular to the plane of the glaſs; and Z the vaniſhing point of lines on the reflected ground plane, which are perpendicular to the interſection of that plane and the glaſs. Then proceed as in the laſt leſſon to find the reflexion of the perpendicular or pole of the pyramid *o d*, by its vaniſhing point Y; and from the perſpective of the ſeat of the pyramid, draw the lines *a* 1, *c* 2, *d* 3, *b* 4, perpendicular to the line of interſection P *v*. To the vaniſhing points Z and W, draw reſpectively 1 Z, 2 Z, &c. and *c* W, *d* W, &c. and the points of interſection *a c b* will give the reflexion of the baſe of the pyramid. Laſtly, join *o a*, *o c*, *o b*, and it will produce the perſpective of the reflexion of the whole pyramid.

If what has been done before be well underſtood, and the law of reflexion attended to, the learner will eaſily ſee the reaſon of the whole operation in theſe two laſt leſſons.

LESSON XLVII.

To take the requiſite Meaſurements for putting an Object in Perſpective.

Operation. Let C be a chair to be put in perſpective, and E the ſpectator's eye. Then, any where betwixt the eye and the object on the ground plane, ſuppoſe the line I N to be drawn to repreſent the interſection of the ground plane with the plane of the picture. Let alſo M L T R repreſent the paper on the drawing-board, on which draw *I N* for the line of interſection. Meaſure the height of your eye P E from the ground plane, and make *IV* equal to it; then draw *VA*, on which take at pleaſure the point C for the centre of the picture. Perpendicular to I N, meaſure P B, to which make C S equal, for the diſtance of the eye from the perſpective plane. Imagine *d* 1, *a* 2 to be lines perpendicular to I N; and meaſure B 1, B 2. Theſe diſtances being ſet off from D towards N give the points *p*, *q*; from which raiſe perpendiculars, and make *p m* equal to *d* 1, and *q n* equal to *a* 2. Laſtly, raiſe *q r* perpendicular to *IN* for the line of elevation; on which take *q o* equal to *b e*, and *q r* equal to *b f*. So will every requiſite be known to find the perſpective of the object by the preceding leſſons.

Remark. If *a b c d* be a right angled figure, and *d* 1 equal to *a* 2, the object will be viewed partially. And if *d* 1, *a* 2 be not equal, the object will be viewed on an angle, and muſt therefore have two points of diminution. If B falls at an equal diſtance between the points 1, and 2, it will produce a central view.

LESSON XLVIII.

To find the Diſtance of the Picture.

As this is a point of the greateſt importance in perſpective, it would be well for every one, who would either draw well themſelves, or be able to judge properly of a piece of drawing, to to be particularly attentive to it; being equally as neceſſary to the ſpeculative connoiſſeur as to the real artiſt. For no one can form a proper judgment of the beauties or defects of either portrait or landſcape, unleſs he knows exactly the centre and diſtance of the picture; as it cannot poſſibly appear to the eye of the ſpectator in any other point of view, agreeable to the painter's intention, as we have before obſerved.

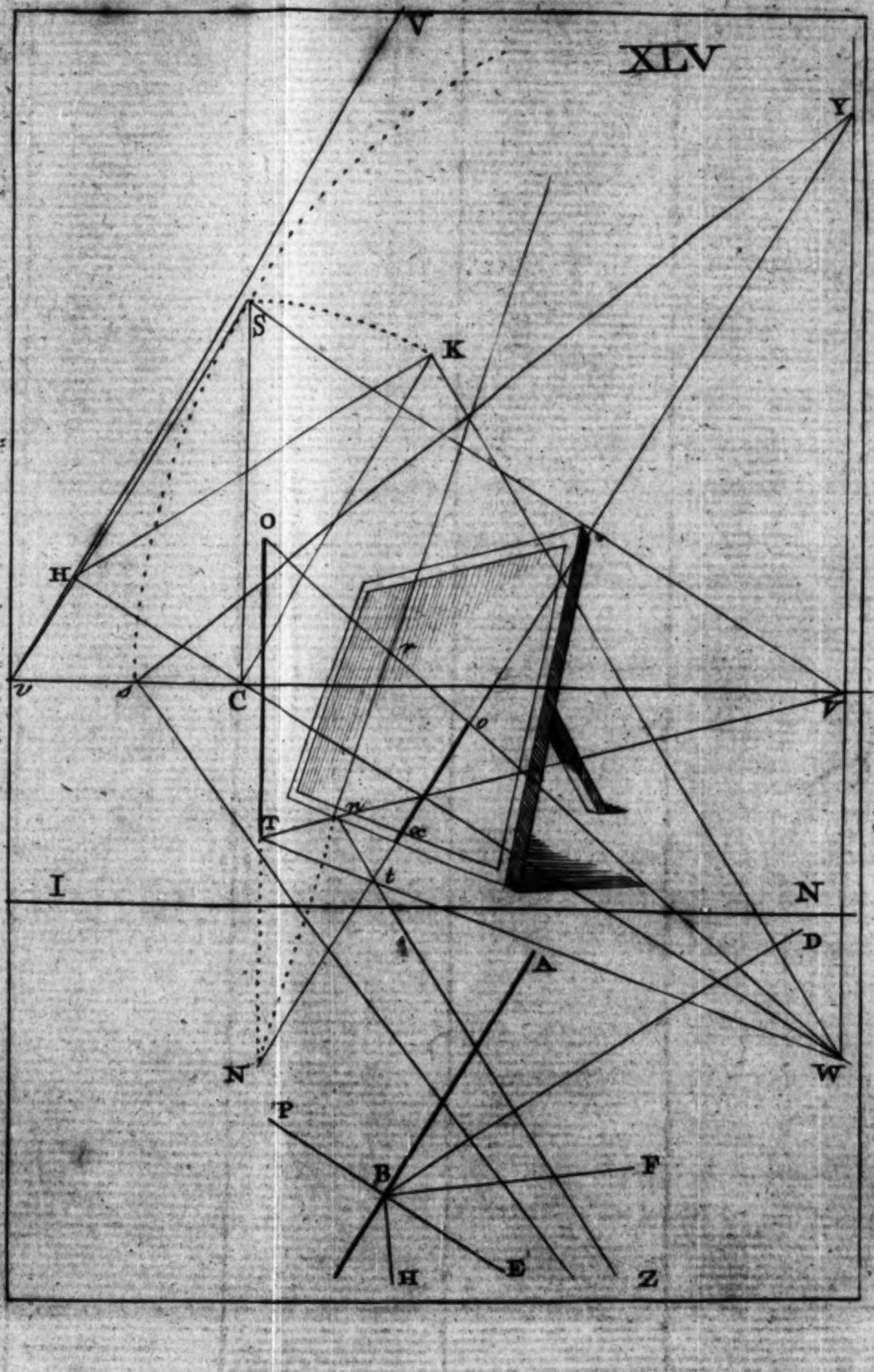
In order then that the learner may have a clear idea of this very eſſential point towards giving a juſt repreſentation of any object, it may not be amiſs to give a ſhort explanation of the ſtructure of the eye, whereby he will the more readily ſee the reaſon why one diſtance of the picture is preferable to another.

In Fig. 5. which repreſents an eye in its juſt proportion, A is called the Cornea Tunica; B the Uvea, in which is a ſmall perforation *c*, called the Pupil, through which the rays of light enter; C the Aqueous Humour; D the Cryſtaline Humour, which is nearly of the form of a double convex lens, being ſuſpended in the middle of the eye by an annulus of muſcular fibres, called the Ligamentum Ciliare, as at *a* and *b*; E the Vitreous Humour; F the coat called Choroides; G the coat called Sclerotica, being a part of the Dura Mater continued about the ball of the eye on its inner part; H the Optic Nerve; and I the Retina, which is a part of the Optic Nerve, expanded over all the interior part of the eye to *a* and *b*. Each of theſe parts have a different refractive power, which all concur towards diſtinct viſion, by forming the image of every external object on the retina, which is by nature appointed the immediate organ of ſight.

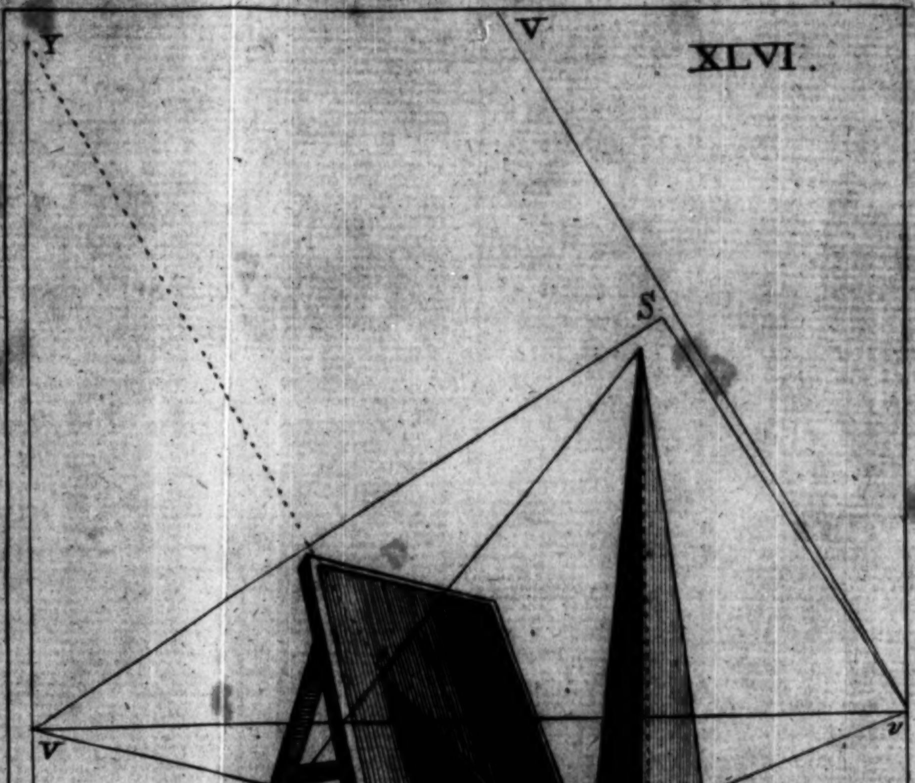
Thus, if A B be an object placed at any diſtance from the eye, a pencil of rays proceeding from any point C will firſt fall on the cornea, and then be refracted by the aqueous humour under it to a point M, in the axis of the pencil behind the retina. The rays then falling on the cryſtaline humour are again refracted, and meet the axis ſtill nearer the retina at N. After which they enter the vitreous humour, where they are again refracted ſo as to meet the axis on the retina at Q. Hence it appears, that the rays of light in flowing from every part of an object A B placed at a proper diſtance from the eye, will form an image on the retina, which is always inverted, or in a contrary poſition to that of the object, becauſe the rays which come from the extreme parts thereof A, B, croſs each other in the middle of the pupil.

Now it has been found from repeated meaſurements of the ſeveral parts of the eye, that the extremities of the cryſtaline humour ſubtend an angle at the pupil of about 90 degrees; conſequently no object can be diſtinctly ſeen by the eye

XLV



XLVI.



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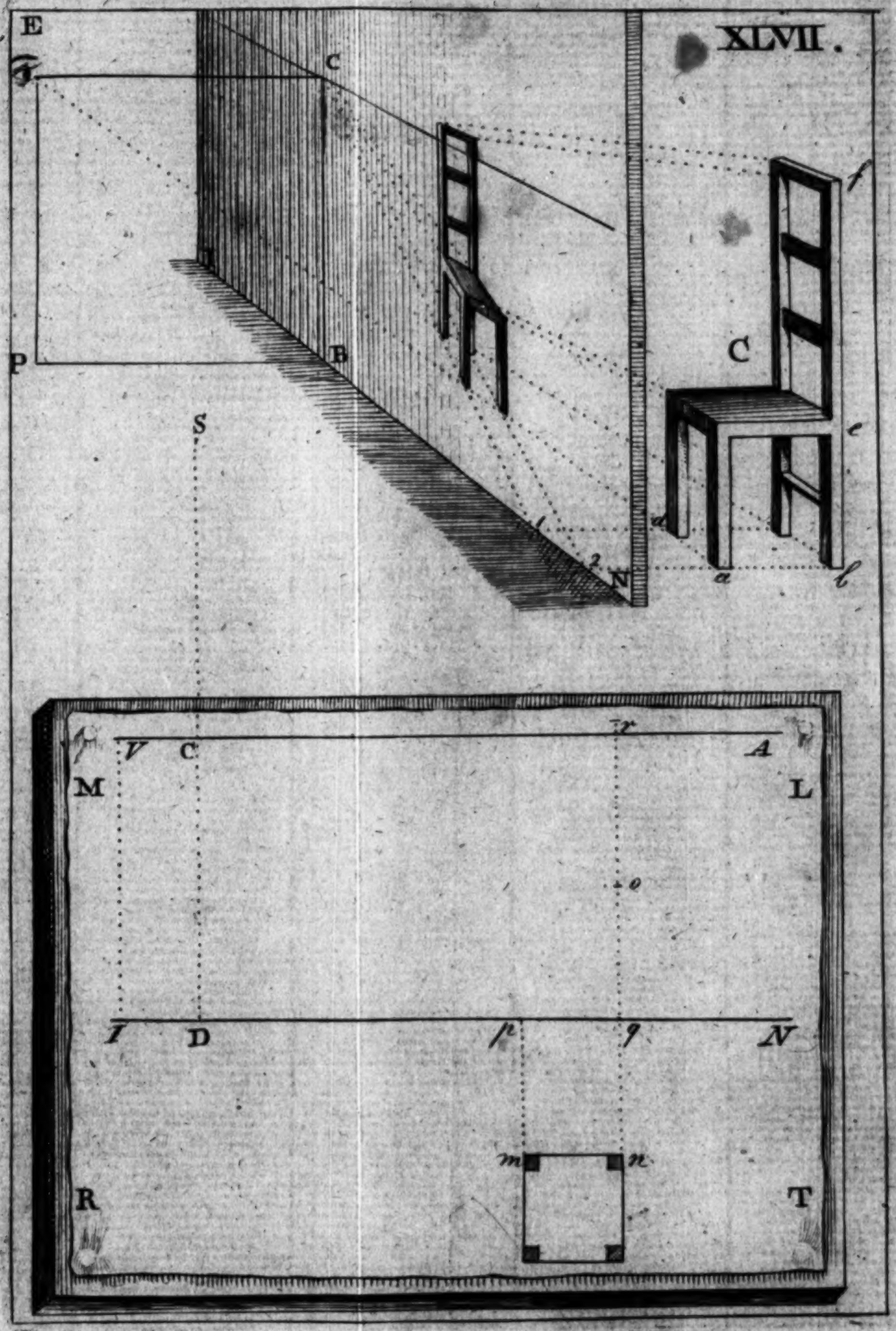


eye under an angle which is greater than that. For if we ſuppoſe the object A B to be brought nearer to the eye as at *A B*, it is plain that the extreme rays *A c*, *B c*, in paſſing through the pupil, will not fall on the cryſtaline humour, but on the ligamentum ciliare, and therefore cannot be duly refracted to the retina. And though the rays proceeding from an object ſubtending an angle at the eye of 90 degrees, may be refracted to the fundus, yet it will not be ſeen diſtinctly; for as the rays have a greater or leſs force as they are leſs or more oblique, it follows, that thoſe rays which paſs through the very middle of the cryſtaline are of moſt efficacy in cauſing viſion, as entering the pupil perpendicularly, and falling in the ſame direction on the retina. Agreeable to this, it is found from obſervation, that any large object is beſt viewed at an angle of about 60 degrees, and ſmaller objects under any angle between that and 30 degrees. If the object be very ſmall it cannot poſſibly be ſeen diſtinctly under a greater angle than 30 degrees, becauſe the refractive powers of the humours will not admit of a leſs diſtance, which in a perfect eye is never leſs than ſix or ſeven inches. Indeed when an object is not far diſtant, there is a power in the eye of lengthening the focal diſtance after the laſt refraction in the vitreous humour, and thereby adjuſting itſelf for diſtinct viſion at different diſtances; but as this is limited to a very narrow ſpace, it cannot ſenſibly affect the above angle for ſmall objects. It has been a matter of diſpute for ſome time among the gentlemen of the faculty, whether this is effected by means of the ligamentum ciliare, in bringing the cryſtaline humour a little nearer the cornea; or, by a power in the optic nerve to alter the convexity of that humour.

But to ſhow the application of this more immediately to the practice of perſpective, let A B C D, Fig 6. be a ſquare ſituated on the ground plane, I N the line of interſection, V A the vaniſhing line, &c. as before. Now if the eye of the ſpectator was at *s*, it is plain he could not ſee the two ends of the ſquare A and B, becauſe the eye could ſee at moſt but ſo much of it as is comprehended under the right angle *c s d*. But if the eye was at *ſ*, it could ſee the ends A and B, becauſe the angle *a ſ b* is a right one. And the eye would ſee the ends A and B better at *S*, and better yet at S, where the angle *a* S *b* is 60 degrees. And from ſuch a diſtance, the objects may be ſeen in perfection; the viſual angle being neither too great nor too little. Therefore when the perſpective plane is imagined to be contiguous to the neareſt part of an object, the diſtance C S muſt be taken ſo large as to make the angle C *b* S not leſs than 45 degrees. But as the perſpective plane may be imagined to ſtand any where betwixt the ſpectator's eye and the object, it is evident that this is not a general rule, though it is given as ſuch in moſt books of perſpective. For it is plain from Fig. 5. that whether the perſpective plane be placed contiguous to the object, as at P E; or any where nearer to the eye, as at *p e*, that the object A B will appear to the ſpectator's eye under the ſame angle A *c* B. But if we ſuppoſe the object to be placed at *p e*, it would ſubtend an angle at *c* greater than a right one; and therefore could not be diſtinctly ſeen by the eye at that diſtance. Hence it appears, that it is not the *diſtance of the picture*, but the *diſtance* *of the object* that muſt be principally regarded in the practice of perſpective.

Now let E F D C, Fig. 6. be a parallelogram placed at a diſtance from the line of interſection, and we ſhall have this general rule for finding the diſtance of the picture, when the object is viewed under a given angle. Draw C H perpendicular to I N, and make the angle H F Z equal to the complement of half the aſſumed angle (which muſt not be greater than 60, nor leſs than 45 degrees, as hath been obſerved before) and Z K will be the true diſtance of the picture, for the object to appear to the ſpectator's eye under the given angle.

THEATRICAL PERSPECTIVE.

As the whole Deſign of a Theatre is founded in Perſpective, it ought to be ſtrictly regulated by the Rules thereof. For if we imagine a tranſparent Plane to be put in the Place of the Proſcene or Curtain, it is evident, that an Obſerver would view the whole Affair tranſacted perſpectively; and therefore, not only the Stage, but alſo the Sides and Ceiling, Proſcenes and Poſtſcenes ſhould be all in regular Perſpective.

Hence it is, that the whole Houſe, internally, is ſo contrived, as to repreſent a perſpective View of an hollow Parallelopipedon, or large Room, about three or four Times as long as it is broad or deep, the Ceiling, Sides, and Floor, all converging by the Rules of Perſpective to the Centre of the Picture at the fartheſt End, which is the remoteſt Part of the Stage.

LESSON XLIX.

To determine the Form and Poſition of the Stage; the Vaniſhing Lines and Centres of the Scenery; and the Situations and Lengths of the Grooves for the ſliding and lateral Scenes.

Operation. Let A B D E be an horizontal plane, F G a line drawn through the middle of it, and F H the height of the eye. Then if H *c* be drawn parallel to F G, it will cut the proſcene or perſpective plane I K L N, in C; which is therefore the centre of the picture, and and V A the vaniſhing line. Now upon this perſpective plane the picture of the ſtage will be repreſented by I N C; the lateral ſcenes or ſides by I K C, N L C; and the aerial ſcene or ceiling by K L C; all which ſhould have a perfect coincidence in the ſpectator's eye with the parts they repreſent. This is effected in the following manner:

Inſtead of the upright plane I L, let another plane I N *a v* be placed on the ſame line I N, and ſo far inclined, that its elevation G *c* may be juſt equal to the height of the eye F H. Then will the end of the inclined plane *v a* exactly coincide to the eye at H with the vaniſhing line V A, and the perſpective ground plane I N C on the proſcene be projected into the large one I N *c*; which is therefore the true form and elevation of the ſtage. The heights of the ſeveral lateral ſcenes, or ſliders, are to be adjuſted from the inclination of the lines K *c*, I *c*; and the diſtances for the grooves may be thus found; Divide the line N C, which is the perſpective of the ground line of the ſtage, into the perſpective equal parts, 1, 1; 2, 2; 3, 3, &c. and from H refer them to the ſide of the ſtage N *c*; then will 1, 1; 2, 2; 3, 3, &c. be the true places and diſtances of the grooves, for the lateral ſcenes to appear in true perſpective to the eye of a ſpectator at H.

The vaniſhing line *V A*, of any poſtſcene *i k l m*, is beſt found by a ſection of the ſtage *c* O G;

c O G; from whence the height of the eye A C, from the bottom of the ſcene, is eaſily determined to any given diſtance from the proſcene O P; being only the difference between the height of the horizontal plane O C, and the perpendicular height of the ſtage A P in that place. By obſerving this rule, there will be a perfect uniformity in the ſcenery; as the vaniſhing line of every poſtſcene will then be in the ſame horizontal plane, and the whole will thereby have a very agreeable and natural appearance to the ſpectators.

It appears from what has been ſaid, that a perſon will ſee a theatrical performance moſt juſtly repreſented, from the fartheſt end of the pit, or in the middle boxes; where every thing will appear to the greateſt advantage. For the eye will then be in the centre of the perſpective plane, and at a proper diſtance from the proſcene, which generally appears there under an angle of about 55 or 60 degrees. The Mancheſter Theatre-Royal is admirably well executed in this reſpect, and does great honour to the judgment

XLVIII

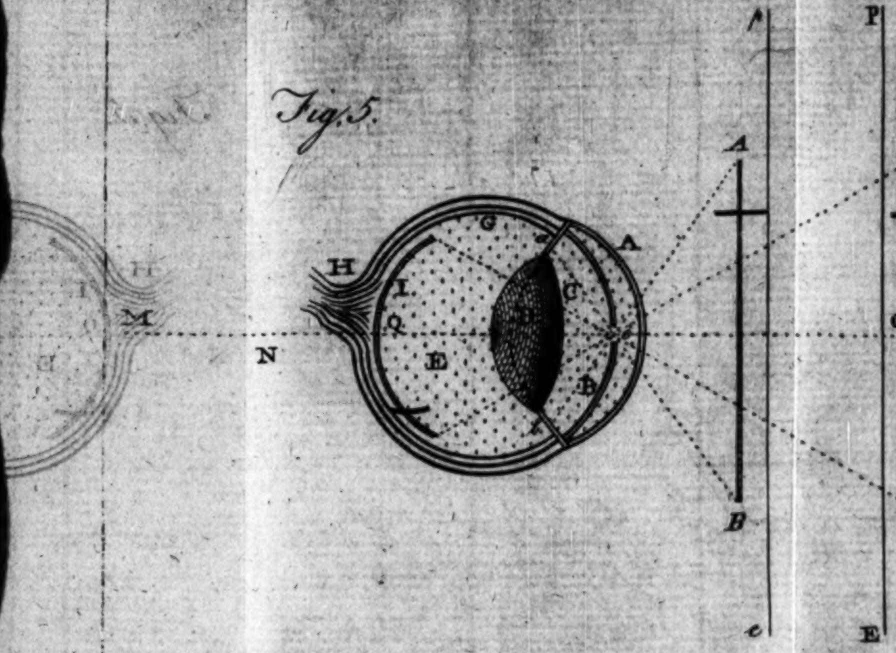


Fig. 5

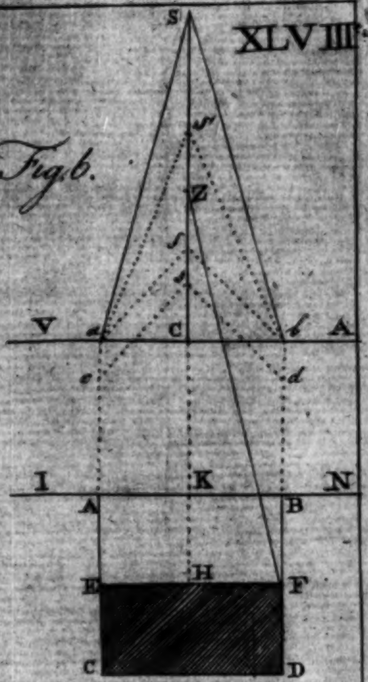
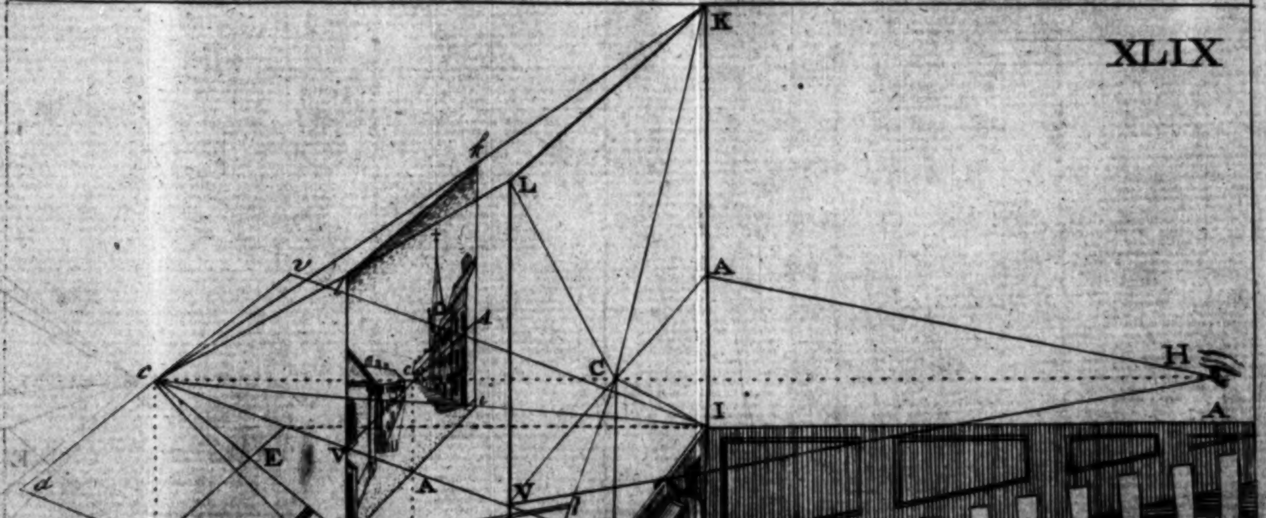


Fig. 6

XLIX



judgment of the projectors. For to the eye of an obſerver ſituated at the farther end of the pit, or in the middle boxes, there is a perfect coincidence in the perſpective, not only of the poſt-ſcenes, but alſo of the ſtage and lateral ſcenes; all uniting in one common point of view. And the diſtance of the front of the middle boxes from the orcheſtra is ſo adjuſted, that the proſcene, or field of view, ſubtends an angle at the eye of the ſpectator of 60 degrees; under which angle we have before ſhown, any large objects will be ſeen in the greateſt perfection.

HORIZONTAL PERSPECTIVE.

This Kind of Perſpective relates chiefly to Ceiling Pieces; which, when properly executed, will make the Objects appear to the Spectator's Eye, as if they were drawn on the Sides of the Room continued upwards. And in order to produce this Deception, nothing more needs be done, than to make the Centre of the Ceiling the *Centre of the Picture*; and the Height of it above the Spectator's Eye the *Diſtance of the Picture*; and then proceed according to the foregoing Directions for Objects ſituated on the Ground Plane.

LESSON L.

To draw a Line upon a flat Ceiling, which ſhall appear, to the Eye of the Spectator, to be perpendicular to the Ground Plane, or parallel to the Sides of the Room.

Operation. Let A B C D be the ceiling, E the place where the perpendicular is to be *apparently* raiſed from, and E F the given height of it; and let C be the centre of the picture, and C S (or C *S*) the given diſtance. Then from E, draw E C; and from F, draw F *S*; and E *f* will be the repreſentation of the perpendicular line, as required.

After the ſame manner may the ceiling be made to appear as the ſides of the room continued upwards, as follows: From each corner of the room to the centre of the picture, draw the lines A C, B C, C C, and D C; then draw D *S*, and from the point of interſection *c*, draw *c b*, *b a*, *a d*, *d c*, reſpectively parallel to the ſides of the ceiling C B, B A, A D, D C. And *a b c d* will be the perſpective ceiling, which will appear as if it were elevated higher by C D than before; and the parts A *a d* D, D *d c* C, C *c b* B, and B *b a* A will have the ſame effect to the eye at S, as if the ſides of the room were actually continued upwards to that height.

LESSON LI.

To draw the Repreſentation of a Window upon a a flat Ceiling, which ſhall appear to the Spectator's Eye, as if it were in the Side of the Room continued upwards.

Operation. Let A B C D be the ceiling of the room, and *a b c d* the perſpective of it. Let alſo E F be the breadth, and E *g* the height of the window; and E *e* the thickneſs of the wall. To the centre of the picture C, draw E C, *e* C, and F *f*; then draw *g S*, and make *m n*, *m n*, *m f*, reſpectively parallel to A D, A B; and it will produce a juſt repreſentation of a window in the ſide of the room, when it is completed as on the oppoſite ſide A B.

If the window be arched at the top, find the points *o t x p* as before; and take *b i* equal to the height of the arch, and draw *i S*. Then make *r v* parallel to B C, *v u* parallel to A B, and *u s* parallel to B C. Laſtly, draw the curved lines *o t*, *x p*, and finiſh the whole according to the figure on the other ſide A D; and it will have the deſired effect to an eye oppoſite to C, and at the diſtance S C. After the ſame manner may cylinders, pilaſters, arches, doors, cornices, domes, human figures, &c. be drawn on a ceiling, ſo as to appear upright, or perpendicular to the ground plane; which is fully exemplified in the ſecond volume.

The practical method of drawing upon ceilings, is to lay down the dimenſions of the ceiling upon paper, and with the height of the room as the diſtance, to finiſh the whole drawing, both with regard to the lines and colouring; then transferring the whole to the ceiling by reticulation, and finiſhing it agreeable to the model.

LESSON LII.

Directions for taking Perſpective Views without any Meaſurements.

Operation. Having fixed your paper on the drawing-board, draw a line to repreſent the baſe line of the picture, or the place on which you ſtand; and about the middle of the paper draw another line parallel to it, for the horizon or boundary of your ſight. Then, in order to facilitate the meaſuring of the diſtances of each part of the view by your eye, draw the neareſt object on the left hand, as church, houſe, tree, &c. as accurately as you can. Having done this, hold the edge of your drawing-board horizontally, ſo as to cut or lie over the extremities of the next diſtant object; and obſerve what part of the object on the fore-ground is there covered by the edge of the board. Then draw, or imagine to be drawn, on the paper, a line parallel to the baſe line and paſſing through the correſponding part of the object already drawn on your paper. Thus proceed till you have got to the moſt remote objects. Then do the ſame with what is to be ſeen on your right hand, and you will have every thing laid down pretty near the truth, both with reſpect to diſtance and proportion. When you have finiſhed your rough draft, which may be done in black lead, Italian chalk, &c. transfer the whole to the paper on which you intend to complete your drawing; and before you begin to out-line or finiſh any part of it, correct the perſpective by obſerving the following directions: 1. All objects, as the ſides of houſes, walls, hedges, walks, avenues, &c. which you obſerved to be parallel to your drawing-board (when held in the before-mentioned poſition) muſt alſo be parallel to the line of interſection or baſe line in the picture. 2. Whatever you obſerved to be perpendicular to the board, muſt diminiſh into the centre of the picture. 3. If an object was ſituated obliquely to the board, lay the edge of a ruler along any line which is horizontal, as the eaves of an houſe, &c.

and mark the point where it cuts the vaniſhing or horizontal line, and that will be the point of diminution, and alſo the vaniſhing point of all other objects which were in the like ſituation in reſpect to the drawing-board. 4. If in producing the ſlant lines of the ſame object, you ſhould find that they do not interſect in the vaniſhing line, it ſhows that you have not been accurate in your rough ſketch, which muſt therefore be corrected by the point of interſection being brought into the vaniſhing line. For it cannot poſſibly happen in that manner, unleſs the object ſtands inclined to the horizon.

When your out-line is correct, begin with the ſhading part: This is beſt performed by ſhading the whole faintly over firſt, and ſo proceeding by degrees of ſhade till you have made the whole agreeable to your eye. By this method you will be better able to judge of the propriety of the ſhades, than by finiſhing it by piece-meal, which a great many practiſe very injudiciouſly. You muſt likewiſe obſerve to make all your light and ſhades fall one way, and let every thing have its proper motion, as trees ſhaken by the wind,

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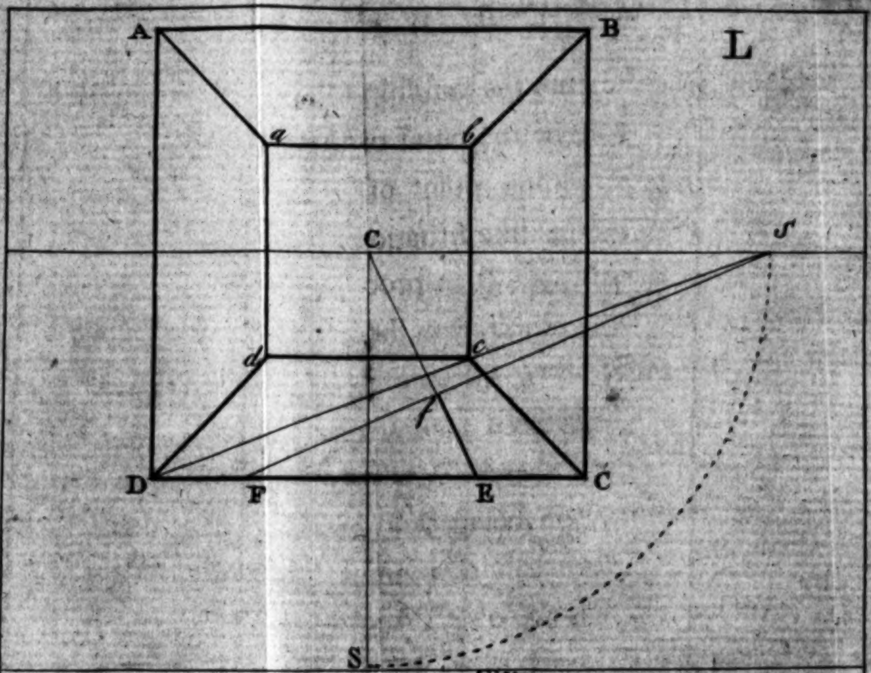

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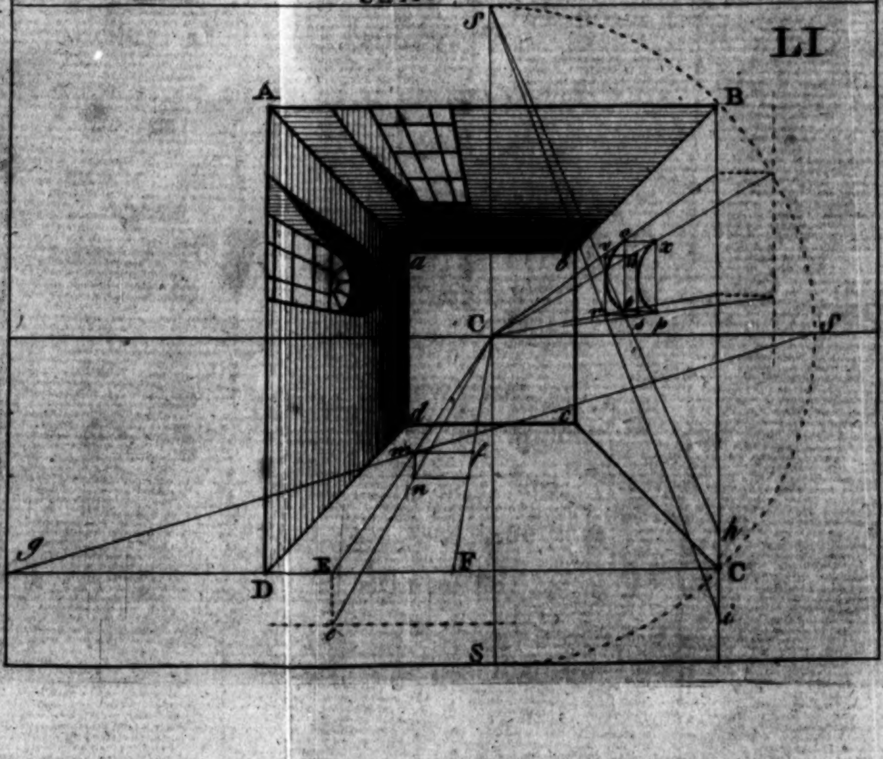

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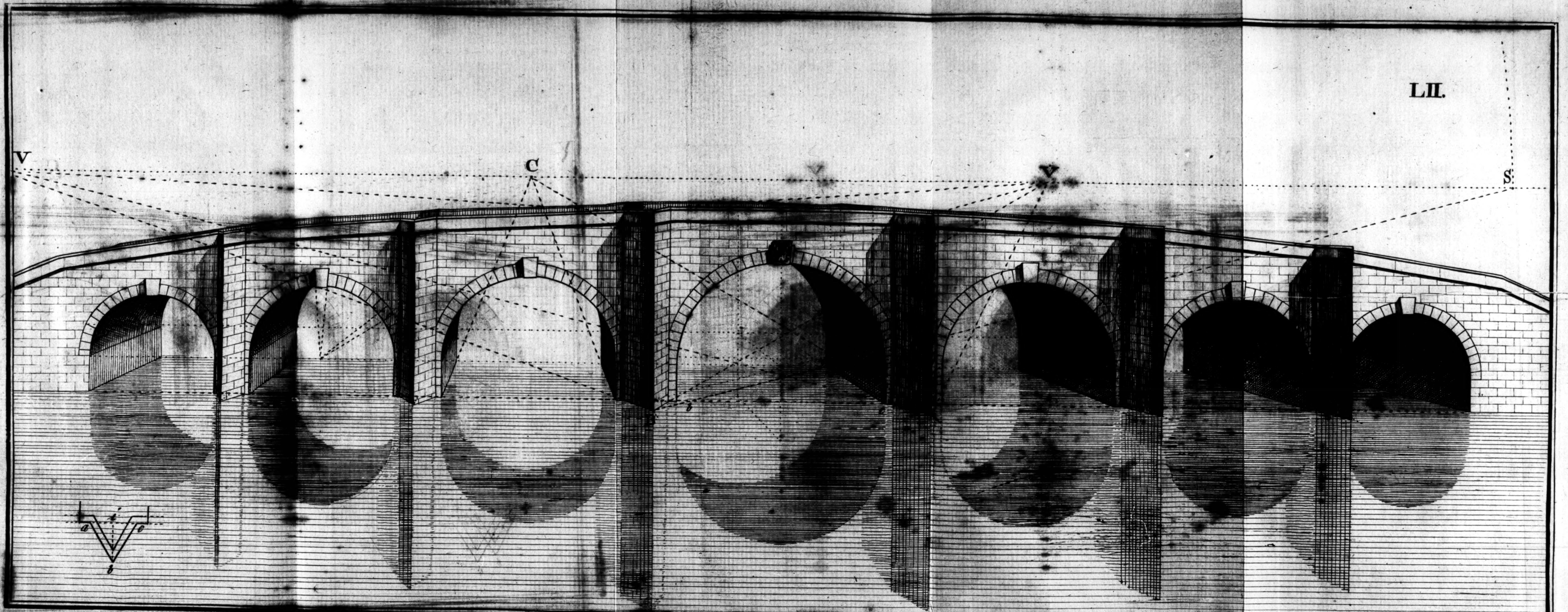

W M Sculp.

the ſmall boughs bending more than the large ones; water agitated by the wind, and daſhing againſt ſhips or boats, or falling from a precipice upon rocks and ſtones, and ſpurting again into the air. The work ſhould always imitate the ſeaſon it is intended to repreſent, and every view have its proper adjuncts; as the farm-houſe ſhould be accompanied with herds of cattle, flocks of ſheep, mills, woods, &c. But to acquire a facility of performing with propriety all the additional graces in a perſpective view, a good copy will be found of far greater ſervice than a verbal deſcription. I know of none more proper for this purpoſe than Collet's landſcapes.

Plate LII. is added for the learner's exerciſe; where C is the centre of the picture; C S the diſtance; *a b c* the ſalient angle of the central piers, *i b* their perpendicular projection, and V, V their vaniſhing points; and *e b* the breadth of the bridge.

THE END OF THE FIRST VOLUME.

Advertiſement.

THE ſecond Volume of this Work will be publiſhed as ſoon as ever the Plates can be got ready; which will occaſion ſome Delay, as they conſiſt chiefly of finiſhed Objects, Views, &c. and are executed in a very maſterly Manner.

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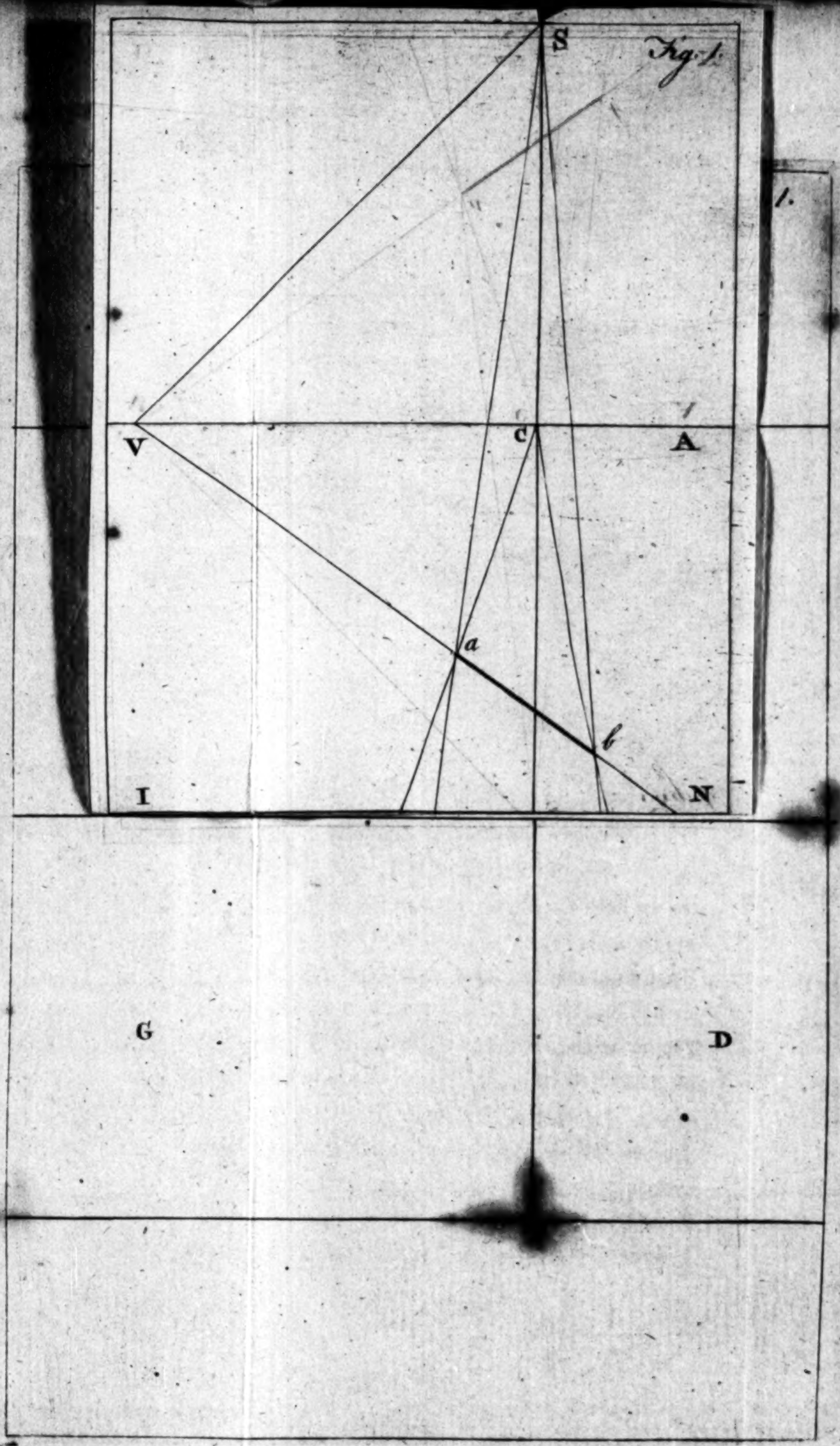

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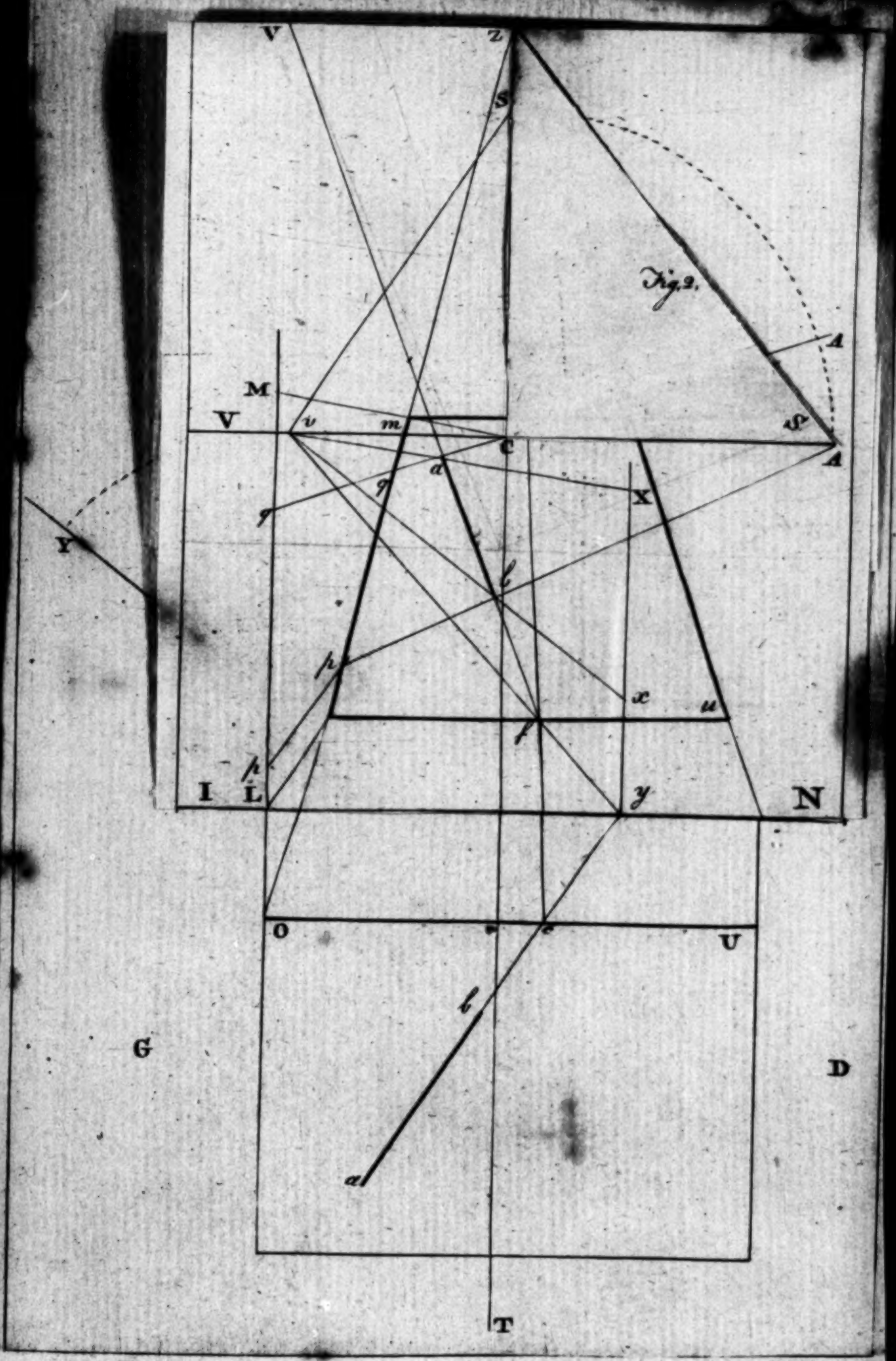
Fig. 2.

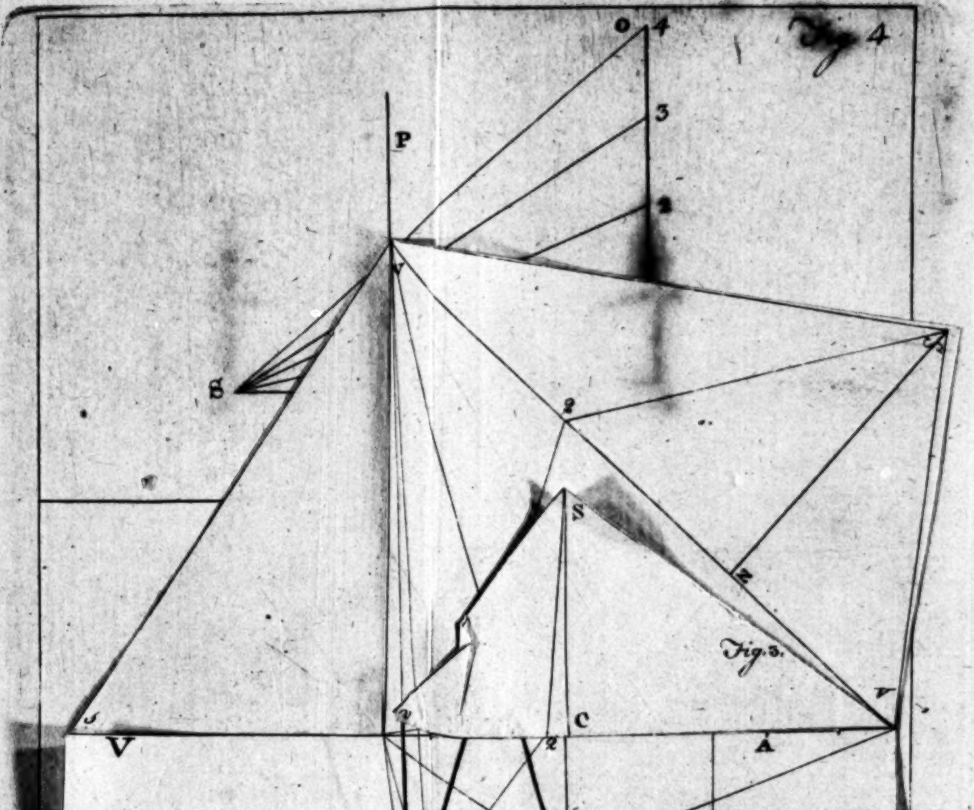
Fig. 4

Fig. 3.

Fig. 3.

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